Synthesis of the Control Algorithm to the Models of Objects with Inertia First Order and Second Order Astatism

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***Abstract*— In this work it is proposed to synthesize the control algorithm for the models of objects with inertia and second order astatism, which are described the dynamics of various technical objects and technological processes. These models of control objects have the double pole in the origin of axes and one negative pole. In order to tune the PID control algorithm to the given model of object, it was synthesised the control algorithm based on the maximum stability degree criteria with iterations. To verify the obtained results of tuning the PID controller, it was done the synthesis of the control algorithm by the polynomial equations method. An example of a system with the respectively model of control object and the controller synthesized according to these methods was computer simulated in the MATLAB software package and it was done the analysis of the system performance. There are highlighted the advantages of the maximum stability degree criteria with iterations by the simplification of the tuning procedure of the PID controller to this model of object.**

Keywords—model of object with inertia and second order astatism; transfer function; control algorithm PID; tuning of the controller parameters; the maximum stability degree criteria with iterations; polynomial equation; system performance

I. Introduction

At the automation of the various technical objects, industrial and technological processes, where their evolution can be approximated by the mathematical models presented as transfer functions with the parameters that depend on the internal properties [1, 2]. According to these properties, the mathematical models attached to these processes will have the respective structure and the corresponding order.

There are a big variety of technical objects (automobile, spacecraft, rocket, telescope, plotter, laser, elevator, nuclear reactor etc. [1, 2]) and technological processes, which are described by the mathematical models with inertia and second order astatism presented by the transfer function in the following form:

$$H(s) = \frac{k}{s^2(Ts+1)} = \frac{k}{Ts^3+s^2} = \frac{k}{a_0s^3+a_1s^2}, \qquad (1)$$

where k is transfer coefficient, T – inertia time constant and $a_0 = T$, $a_1 = 1$ present the general coefficients.

The presence in the model of object (1) the second order astatism increase the difficulty of tuning controller to this model of object. The widely used methods for tuning the PID controller such as Ziegler-Nichols method, frequency method etc. can not be applied for so kind of model of objects [1, 3-7]. In this paper, it was proposed to use the maximum stability degree (MSD) method with iterations for tuning the PID control algorithm [8-10].

II. The Algorithm for Tuning the PID Controller

In this study, the automatic system it is described by the block scheme presented in the Fig. 1, that consists from the controller with transfer function $H_R(s)$ and model of object with transfer function $H(s)$.

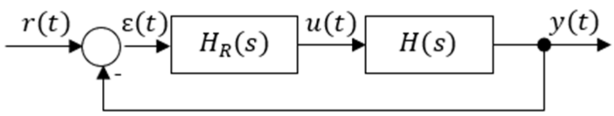


Fig. 1. Structural scheme of the control system.

The control algorithm PID is described by the following transfer function:

$$H_R(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \qquad (2)$$

where k_p, k_i, k_d are the tuning parameters of the PID controller.

Applying the maximum stability degree criteria, there are presented the analytical expressions for determination of the tuning parameters for the model of object (1) in the form [8-10]:

$$4a_0J - a_1 = 0, \qquad (3)$$

$$k_d = \frac{1}{2k}(-12a_0J^2 + 6a_1J) = \frac{3a_1^2}{8ka_0} = f_d(J), \qquad (4)$$

$$k_p = \frac{1}{k}(4a_0J^3 - 3a_1J^2) + 2k_dJ = \frac{a_1^3}{16ka_0^2} = f_p(J), \quad (5)$$

$$k_i = \frac{1}{k}(-a_0J^4 + a_1J^3) - k_dJ^2 + k_pJ = \frac{a_1^4}{256ka_0^3} = f_i(J). \; (6)$$

For the case of known the values of the model of object (1), it is determinate the value of optimal stability degree J from equation (3) and according to the expressions (4)-(6), there are calculated the optimal parameters k_p, k_i, k_d of the PID algorithm.

In some cases, for optimal values k_p, k_i, k_d of the PID controller obtained from the expressions (3)-(6), the performance of the automatic system is not satisfied. In this case it is proposed to vary $J = 0 \cdots \infty$ as an independent variable and it is calculated and constructed the curves (4) – (6) $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$.

From these curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ there are chosen the sets of values of the tuning parameters of the PID controller $J_i - k_{pi}, k_{ii}, k_{di}$ and further is done the computer simulation and there are obtained the transient response based on which it is determine the highest performance of the control system, which would satisfy the imposed performance to the system.

For an example of a model of control object (1) with the transfer coefficient k and time constant T, there are analysed the procedures of tuning the PID controller by the MSD criteria with iterations.

There are analysed the performances and robustness of the automatic system for the case when on the system it is applied the perturbance signal by the type step signal and at the variation by ± 50% of the k and T parameters from the nominal value of the model of object.

Example. It is considered to be known the values of the model object for the two cases:

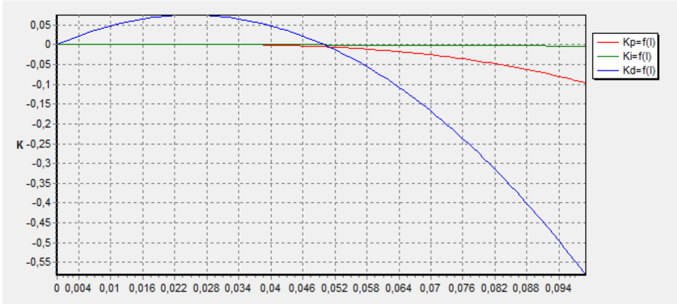
1) transfer coefficient $k = 0.5$, time constant $T = 10$ s;
2) transfer coefficient $k = 0.5$, time constant $T = 0.1$ s.

There are imposed the performance to the automatic system as: steady state error $\varepsilon = \pm 5$ %, rising time $t_c = 2$ s, settling time $t_r = 10\ s$ and overshoot $\sigma = 10$ %.

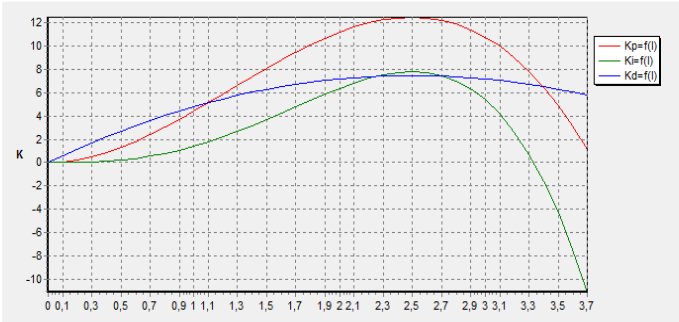
For both variants of models $T = 10$ and $T = 0.1$ it is determinate the optimal degree from (3) and it is calculated the optimal values of the parameters k_p, k_i, k_d by the expressions (4)-(6), which are given in the Table I, rows 1 and 3 respectively.

The curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ were constructed for both cases of object models (Fig. 2 *a*, *b*) and were analysed the sets of values $J_i - k_{pi}, k_{ii}, k_{di}$ from these curves, it was determinate the numerical values of the tuning parameters k_p, k_i, k_d for which there were obtained the highest performances, which are given in the Table I, rows 2 and 4 respectively.

It was done the computer simulation of the control system with PID controller tuned by the MSD criteria and the obtained transient processes are presented in the Fig. 3 *a*) (for $T = 10$, curve 1 – MSD criteria analytical form, curve 2 – the MSD criteria with iterations), and the performance are given in the Table I, rows 1 and 2, in the Fig. 3 *b*) ($T = 0.1$, curve 1 – the MSD criteria analytical form, curve 2 – the MSD criteria with iterations) and the performance are given in the Table I, rows 3 and 4.



a) $k = 0.5, T = 10\ s$.



b) *k*=0.5, *T* = 0,1 s.

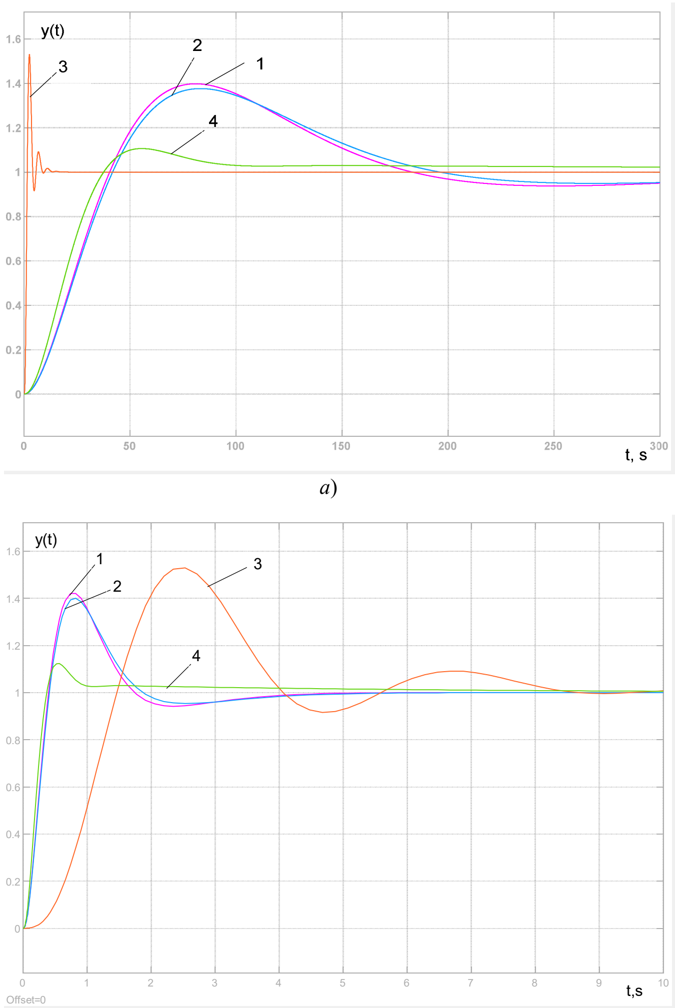
Fig. 2. Dependences $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$.

TABLE I. TUNING PARAMETERS AND AUTOMATIC SYSTEM PERFORMANCE

No	Method	Tuning parameters				Performance of the system			
		J	**k_p**	**k_i**	**k_d**	**t_c, s**	**σ, %**	**t_r, s**	**λ**
1	MSD	0.025	0.0012	$7.8125 \cdot 10^{-6}$	0.075	37.91	40.5	285	2
2	MSD	0.029	0.0011	$6.3411 \cdot 10^{-6}$	0.073	39.19	38.3	170	1
3	MSD	2.5	12.5	7.8125	7.5	0.38	43.9	2.7	2
4	MSD	2.9	11.437	6.3411	7.308	0.39	41.4	1.6	1
5	MP					2.21	33.6	8.6	1
6	MP					1.41	53.1	7.7	3
7	PO		0.0002	$8.52 \cdot 10^{-8}$	0.115	25.1	10.6	371	1
8	PO		2.41	0.085	11.52	0.251	10.6	3.71	1

For the system with the controller tuned according to the maximum stability degree criteria with iterations the settling time is by 1.68 times and overshoot by 1.64 times reduced than the control system tuned by the analytical MSD criteria.

For comparison, the PID controller was tuned by the parametrical optimization from MATLAB. The obtained transient responses are presented in the Fig. 3 *a*) (for $T = 10$, curve 4) and the performance are given in the Table I, row 7, in the Fig. 3 *b*) ($T = 0.1$, curve 4) and the performance are given in the Table I, row 8.



a)

b)

Fig. 3. Transient responses of the control system

III. Synthesis of the Control Algorithm by the Polynomial Method

For comparison of the obtained results of tuning the PID controller to the model of object (2) by the maximum stability degree criteria it was proposed to use the polynomial equation method for synthesis the controller [5, 6]. According to this method the transfer function of the model of object (1) is presented in the following form:

$$H(s) = \frac{k}{s^2(Ts+1)} = \frac{B^-(s)B^+(s)}{A^-(s)A^+(s)}, \tag{7}$$

where $B^-(s)$, $A^-(s)$ are polynomials with left zeros, and $B^+(s)$, $A^+(s)$ - are polynomials with right zeros, and neuters. If polynomials $B(s)$ and $A(s)$ don't contain the left zeros, than $B^-(s)$ and $A^-(s)$ are equal with constants, but if they don't contain positive zeros than $B^+(s)$ and $A^+(s)$ are admited to be equal with one. In this case $B^-(s) = B^+(s) = k$, $A^-(s) = Ts + 1$, $A^+(s) = s^2$. The degrees of the polinomial are denoted by $n_{B^-} = n_{B^+} = 0, n_{A^-} = 1, n_{A^+} = 2$.

The transfer function of the controller is presented in the following form:

$$H_R(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{Q(s)}{P(s)}, \tag{8}$$

where polynomials $M(s)$ and $N(s)$ are determinate from polynomial equation:

$$B^+(s)M(s) + A^+(s)N(s)s^r = A(s), \tag{9}$$

s^r – is astatism by degree r.

The unknown coefficients of the polynomials $M(s)$ and $N(s)$ are determined from the system of algebraic equations, which is obtained by the equaling the coefficients from the same powers of s on both sides of the polynomial equation (9). The obtained system of equations has solutions only if it is satisfied the condition of solving, expressed by the ratio of the polynomials degrees.

In order to determine the degree of the characteristic polynomial of the control system physically realizable according to the method, the conditions are used:

$$n_A \leq n_M + n_N + 1,\ 1 + n_M \leq n_N. \tag{10}$$

Based on the model of object with astatism second order and for satisfying the condition (10), from considerations that transfer function (8) of the controller to be of low degree, there are chosen the minimal polynomials degrees $n_M = 1$, $n_N = 2$ and it is determinate the order of the characteristic polynomial:

$$n_A \leq n_M + n_N + 1 = 1 + 2 + 1 = 4 \tag{11}$$

The steady state error of the system $\varepsilon = 0$, if the characteristic polynomial of the closed system of degree is $n_A = 4$, than it will have the multiple roots $p = -1$ in the form [5-7]:

$$A(s) = (s+1)^4 = s^4 + 4s^3 + 6s^2 + 4s + 1. \tag{12}$$

In automatic systems, in which the characteristic polynomial has the multiple roots, the aperiodic transient response is the fastest.

There are constructed the polynomials with unknown coefficients in the form:

$$M(s) = m_0 s + m_1,\ N(s) = n_0 s^2 + n_1 s + n_2. \tag{13}$$

It is determinate the polynomial equation (9):

$$k(m_0 s + m_1) + s^2(n_0 s^2 + n_1 s + n_2) = (s+1)^4.$$
$$n_0 s^4 + n_1 s^3 + n_2 s^2 + k m_0 s + k m_1 =$$
$$= s^4 + 4s^3 + 6s^2 + 4s + 1. \tag{14}$$

It is obtained the system of algebraic equations from (14), equalling the coefficients from left and right side next to the same powers of s, from which the unknown coefficients of the polynomials are determined (13): $n_0 = 1, n_1 = 4, n_2 = 6, m_0 = 8, m_1 = 2$.

It is determined the transfer function of the controller (14):

$$H_R(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{(Ts+1)(m_0s+m_1)}{k(n_0s^2+n_1s+n_2)} =$$
$$= \frac{(10s+1)(8s+2)}{0.5s^2+2s+3} = \frac{80s^2+28s+2}{0.5s^2+2s+3} = \frac{Q(s)}{P(s)}, \quad (15)$$

$$H_R(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{(Ts+1)(m_0s+m_1)}{k(n_0s^2+n_1s+n_2)} =$$
$$= \frac{(0.1s+1)(8s+2)}{0.5s^2+2s+3} = \frac{0.8s^2+8.2s+2}{0.5s^2+2s+3} = \frac{Q(s)}{P(s)}. \quad (16)$$

The computer simulation of the automatic system was done for this case, with controller tuned by the polynomial equation method and the transient responses are presented in the Fig. 3 *a*) (for $T = 10$, curve 3), the performance are given in the Table I, row 5 and in the Fig. 3 *b*) ($T = 0.1$, curve 3) and the performance are given in the Table I, row 6.

For the system with PID controller tuned by the maximum stability degree criteria, by the increasing of T with 50 % $T^+ = 15$ the rise time t_c is increased by 1.1 times and settling time t_r is reduced by 1.08 times, but overshoot is reduced by 1.21 times; with the reduction the T with 50 % $T^- = 5$ the rise time t_c remains unchanged, the settling time t_r is reduced by 1.56 times, but overshoot is reduced by the 1.36 times. By the increasing the k with 50 % $k^+ = 0.75$ the rise time t_c and t_r is reduced by 1.27 times and by 2.14 times and the overshoot is reduced by 1.15 times, and by reducing the k with 50 % $k^- = 0.25$ the rise t_c and setling time t_r increases by 1.63 times and by 2.14 times, but overshoot increases by the 1.28 times.

For the system with controller tuned by the polynomial method by the increasing the T with 50 % $T^+ = 15$ the rise time t_c and settling time t_r are reduced by 1.3 times and 1.91 times, the overshoot is increased by 1.19 times, but with reducing the T by 50 % $T^- = 5$ the rise time t_c and setling time t_r are reduced by 1.52 times and 2.06 times and overshoot remains without changes.

By the increasing the k by 50 % $k^+ = 0.75$ the rise time t_c and setling time t_r are decreased by 1.31 times and 1.49 times, respectively the overshoot is increased by 1.1 times, but with decrease the k with 50 % $k^- = 0.25$ the rise time t_c and setling time t_r are incresed by 1.63 times and 2.24 times, overshoot is increased by 1.09 times.

For the case, when constant time $T = 10$ s and controller tuned by the MSD criteria, the performance of the system - the rise time t_c and setling time t_r are much lower, than the same performances of the system with the controller tuned according to the polynomial method, and the overshoot is by 7% higher and conversely for the case when $T = 0.1$ and controller tuned by the MSD criteria the performance - rise time t_c and settling time t_r are higher by 3.71 times and 2.82 times, than the same performance of the control system with controller tuned by the polynomial method and the overshoot is by 9% lower.

In case of doubling the $k = 1$ and $T = 0.1$, the performance of the system with controller tuned by the MSD criteria is kept performance unchangeable, but the performance – rise time t_c and settling time t_r of the system with controller tuned by the polynomial method increase and overshoot is reduced.

At the action of the perturbation signal $p(t) = \pm 1(t)$ on the object with $T = 10$ and $T = 0.1$ the transient response of the system is restored during the settling time.

IV. Conclusions

Analysing the obtained results it can be concluded:

- For big time constants, the performance– rise time t_c and setling time t_r of the automatic system with controller tuned by the maximum stability degree criteria are much lower in comparison with performance of the automatic system with controller tuned by the polynomial method, but the overshoot differs by 7 %.

- For small time constants the performance – rise time t_c and setling time t_r of the automatic system with controller tuned by the MSD criteria are much higher than the performance of the automatic system with controller tuned by the polynomial method.

- In case of doubling the $k = 1$ and $T = 0.1$ the performance of the control system with controller tuned by MSD criteria are kept unchangeable, but the performance – rise time t_c and setling time t_r of the system with controller tuned by the polynomial method are increased and overshoot is reduced.

- The overshoot for the analysed system varies from 30% to 53 %.

- The automatic system with $T = 0.1$ and the controller tuned by the MSD criteria with iterations ($p = -0.367$) is more robust by 3.67 times, than the system with the controller tuned by the polynomial method ($p = -0.1$) and by 8.19 times than the system with the controller tuned by the parametrical optimization ($p = -0.0448$) from MATLAB.

Acknowledgment

This work was supported by the project 20.80009.5007.26 „Models, algorithms and technologies for the control, optimization and security of the Cyber-Physical systems”.

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