Synthesis of the PID Control Algorithm for the Models of Objects with Second Order Astatism

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***Abstract*—In the paper is highlighted the industrial objects as cars, spacecraft, telescopes, plotters, lasers, elevators, etc., which are described by the mathematical model with double astatism. These models of control objects have a double pole in the origin of the coordinate axes. In order to tune the PID controller to the model of object with double astatism, it was elaborated the tuning algorithm based on the maximum stability degree method with iterations. It was done the computer simulation of the automatic control system with the respectively model of object and PID controller and it was done the analysis of the obtained performance. The advantages of the maximum stability degree method with iterations were highlighted by the reducing calculations and time, which lead to the procedure simplification of the PID controller tuning.**

***Keywords*—model of the control object with double astatism; transfer function; PID control algorithm; tuning of the controller; the maximum stability degree method with iterations; system performance**

I. INTRODUCTION

In the practice of automation there are a large variety of technical objects, industrial and technological processes, which require the automatic control [1, 6]. These control objects are characterized by the main internal properties as: inertial-capacitive, self-preservation of the state and time delay, which generally determine the mode of operation of the process. Depending on these properties, the mathematical models attached to these processes will have the respective structure and the corresponding order.

In the practice of automation, the most used are the mathematical models in the form of transfer functions, obtained based on the identification procedures applied to the experimental data obtained from the control object [4-6].

There are many technical objects such as car, the spacecraft, the telescope, the plotter, the laser, the elevator, linear drives, etc. [1, 6], which are described by the mathematical models with double-astatism presented with the transfer function with second-order astatism:

$$H(s) = \frac{1}{Ts^2} = \frac{k}{s^2}, \tag{1}$$

where T presents the property of the industrial object, or the technological process and expresses the integration constant time with the dimension s^2 and $k = 1/T$.

The presence of the model of the control object (1) as a double integrator increase the difficulty of the tuning the controller to these models. Several methods for tuning controllers such as the Ziegler-Nichols method, the poles-zero method, the frequency method, etc. cannot be applied [1, 6]. In this paper it was proposed to use the maximum stability degree method (MSD) with iterations for synthesis of the PID controller [7-10].

II. THE ALGORITH FOR TUNING THE PID CONTROLLER

In the study, it is used the block scheme of the control system that is consisting of controller with transfer function $H_R(s)$ and model of control object with transfer function $H(s)$ given in the Fig. 1.

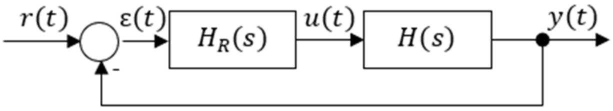


Fig. 1. Structural scheme of the control system.

The PID control algorithm is described by the following transfer function:

$$H_R(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \tag{2}$$

where k_p, k_i, k_d are the tuning parameters of the PID controller.

Applying the maximum stability degree method with iteration, there are presented the calculation expressions of the tuning parameters of the PID controller to the model of object (1) in the following form [7-10]:

$$k_d = 3TJ = \frac{3J}{k} = f_d(J,T) = f_d(J), \tag{3}$$

$$k_p = 3TJ^2 = \frac{3J^2}{k} = f_p(J,T) = f_p(J), \tag{4}$$

$$k_i = TJ^3 = f_i(J,T) = f_i(J). \tag{5}$$

The tuning algorithms P, I, PD, PI can not be applied to the model of object (1).

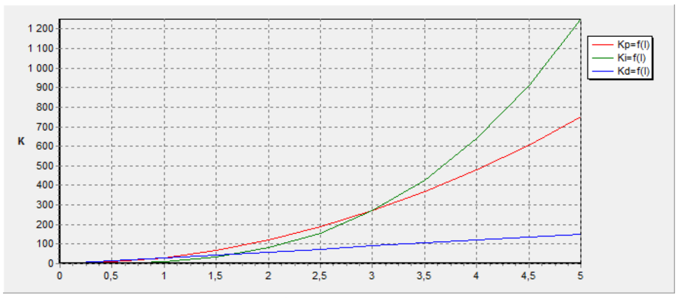
For the case of variation the value $J = 0 \dots \infty$ as independent variable, there are calculated and constructed the curves (3) – (5) $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ for the PID controller.

On these curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ there are chosen the value sets of the tuning parameters of the PID controller $J_i - k_{pi}, k_{ii}, k_{di}$ and further is done the computer simulation and obtained the transient responses of the control system based on which there are determine the highest performance of the system that satisfy the imposed conditions.

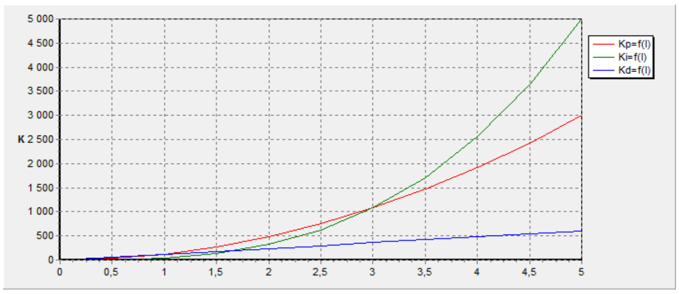
For an example of model of object (1) with $T = 10, 40, 90$, there are analysed the procedures of tuning the PID controller by the maximum stability degree method with iterations and analysed the performance and robustness of the system for the case when is applied the perturbance signal step-type and to the variation of time constant with $\pm 50\%$ from the nominal value.

Example. It is considered to be known the parameters of the model of object $T = 10, 40, 90$. There are imposed the performance to the automatic control system: steady state error $\varepsilon = \pm 5\%$, rise time $t_c = 0.2$ s, settling time $t_r = 1\ s$ and overshoot $\sigma = 10\%$.

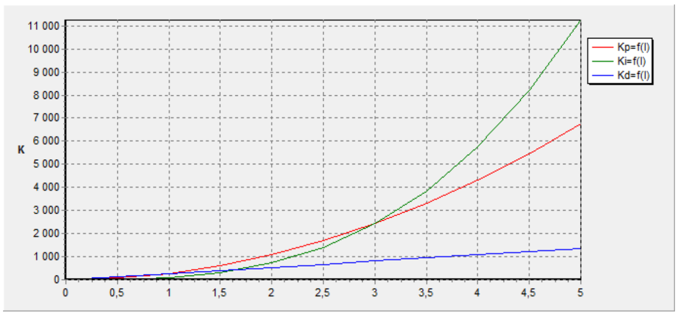
For the values $T = 10, 40, 90$ according to the relations (3)-(5), varying J, it was obtained the dependencies $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ (Fig. 2, *a*, *b*, *c*). Based on these curves, there were chosen the tuning parameters according to the stability degree value $J_i - k_{pi}, k_{ii}, k_{di}$, that are presented in the Table I. The control system with PID controller was simulated in MATLAB and for each case there are obtained the transient processes presented in the Fig. 3, *a, b, c* and the performance of the control system are given in the Table I. The numbering of the curves corresponds with the numbering from the Table I.



a) $T = 10$ s.

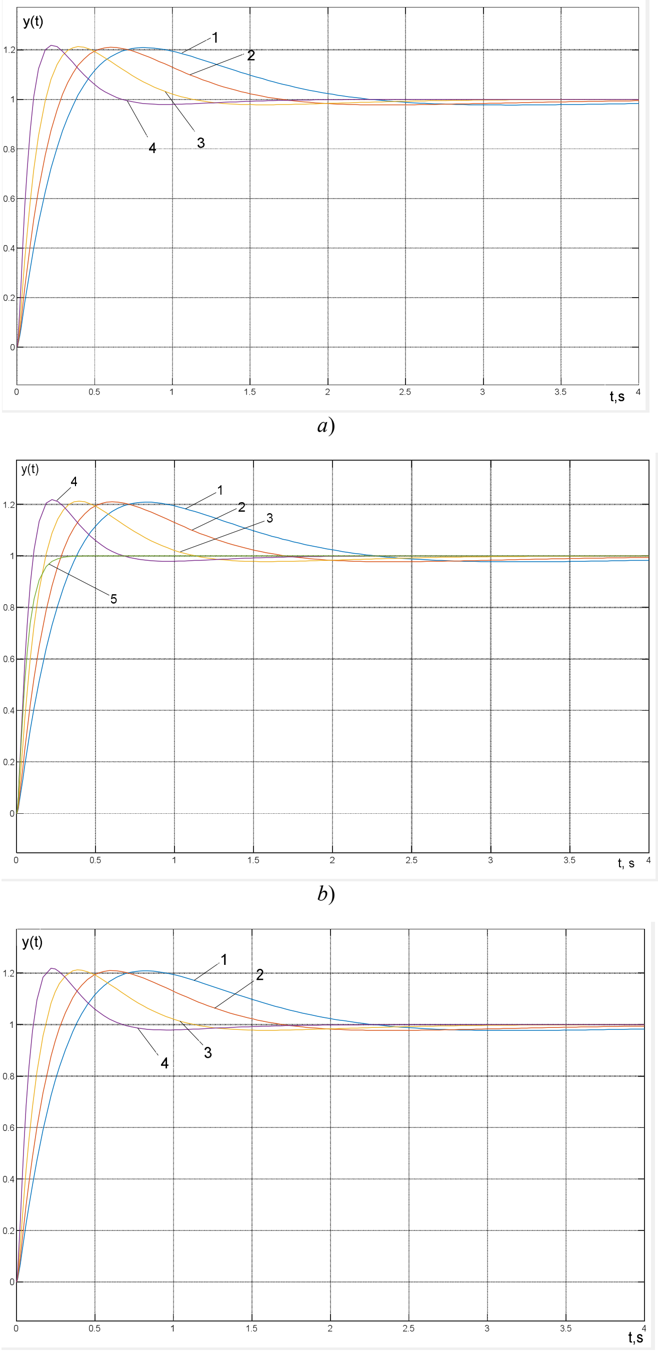


b) $T = 40$ s.



c) $T = 90$ s.

Fig. 2. Dependences $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$.



a)

b)

c)

Fig. 3. Transient processes of the control system.

TABLE I. TUNING PARAMETERS AND AUTOMATIC CONTROL SYSTEM PERFORMANCE

Val. *T*	**No**	**Tuning parameters**				**Performance of the system**			
		J	k_p	k_i	k_d	t_c, **s**	**σ, %**	t_r, **s**	**λ**
10	1	1.5	67.5	33.75	45.0	0.33	21.4	1.7	1
	2	2.0	120.0	80.00	60.0	0.24	21.0	1.3	1
	3	3.0	270.0	270.0	90.0	0.15	22.3	0.8	1
	4	5.0	750.0	1250.0	150.0	0.09	24.0	0.5	1
40	1	1.5	270.0	135.0	180.0	0.33	21.4	1.7	1
	2	2.0	480.0	320.0	240.0	0.24	21.6	1.3	1
	3	3,0	1080.0	1080.0	360.0	0.15	22.3	0.8	1
	4	5.0	3000.0	5000	600.0	0.08	24.0	0.5	1
90	1	1.5	607.0	303.75	405.0	0.33	21.4	1.7	1
	2	2.0	1080.0	720.0	540.0	0.24	21.6	1.3	1
	3	3.0	2430.0	2430.0	810.0	0.15	22.3	0.8	1
	4	5.0	6750.0	11250	1350.0	0.08	21.9	0.5	1

From the analysis of the data from the Table I for the values of the time constant $T = 10, 40, 90$ for the same values of the stability degree J, there are obtained the different tuning parameters, but the system ensures the same performances. For the settling time $t_r = 1$ and using the formula for calculation the stability degree [2-4]:

$$t_r \leq \frac{3}{J}, \tag{6}$$

it is calculated the value of the stability degree:

$$J = \frac{3}{t_r} = \frac{3}{1} = 3. \tag{7}$$

For the value $J = 3$ and $T = 10, 40, 90$ from the Table I for the tuning values k_p, k_i, k_d of the PID there are presented the respectively characteristic equations:

$$Tp^3 + k_d p^2 + k_p p + k_i = 0, \tag{8}$$

$$10p^3 + 90p^2 + 270p + 270 = 0,$$

$$40p^3 + 360p^2 + 1080p + 1080 = 0, \tag{9}$$

$$90p^3 + 810p^2 + 2430p + 2430 = 0.$$

Analysing the algebraic equations (9), it is observed that the second equations is a multiple of 4 and the third equation is a multiple of 9, and it can be concluded that these equations have the same solution and their roots are:

$$p_{1,2} = -2.9961 \pm j0.0047, p_3 = -3.0126.$$

At the action of the perturbation signal $p(t) = +1(t)$ to the control object with nominal value $T = 40$, the transient process of the system becomes ideal and under the action of the perturbation signal $p(t) = -1(t)$ the settling time rise with 1.1 times and overshoot with twice.

It was observed, that by the increasing the value T with 50%, $T^+ = 40 + 20 = 60$, than the settling time t_r rise twice and overshoot with 1.32 times, and by decreasing the value of T with 50 %, $T^- = 40 - 20 = 20$, than the settling time t_r was reduced by 31 % and overshoot σ is reduced with 40 %.

It was found that the tuning parameters $k_p = 0$ and $k_i = 0$ are realized only with component D and its variation of $k_d = 2 \dots 600$ and than the settling time varies $t_r = 59.87 \dots 0.16$ s and transient process of the system is aperiodic (Fig. 3, *b*, curve 5 for the case $T = 40, J = 3$). For the transient process – curve 5, the all imposed performance is satisfied.

For the comparison of maximum stability degree method with iterations with another tuning methods, there are proposed to use one of the optimization methods – the genetic algorithm. This algorithm is based on the idea of natural selection, where the fittest individuals are selected for reproduction in order to produce offspring of the next generation. In this case, the fitness function was settled based on the imposed performance to the system. According to the genetic algorithm, using MATLAB there are calculated the tuning parameters of the PID controller to the model of object with double astatism, Table II.

TABLE II. TUNING PARAMETERS AND AUTOMATIC CONTROL SYSTEM PERFORMANCE

Val. *T*	**No**	**Iterat**	**Tuning parameters**			**Performance of the system**			
			k_p	k_i	k_d	t_c, **s**	**σ, %**	t_r, **s**	**λ**
10	1	23	27.2	18.54	54.14	0.32	9.72	4.4	1
40	2	84	85.81	15.55	163.75	0.41	10.02	7.8	1
90	3	14	100.85	83.32	232.45	0.59	20.6	17.1	1

In the Fig. 4 there are presented the obtained transient processes for the case of tuning PID controller by the genetic algorithm. The numbering of the curves corresponds with the numbering from the Table II.

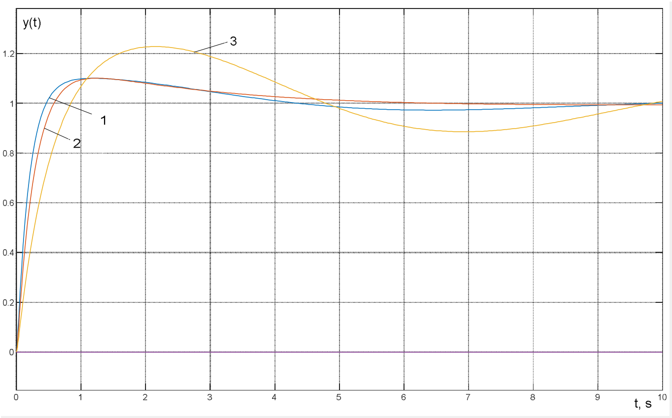


Fig. 4. Transient processes of the control system.

From the analysis of the performance of the automatic control system with PID controller tuned by the genetic algorithm, it was observed that with increasing of the numerical value of the inertia - T it is lead to the moderate increase of the rise time from 1.0 to 1.79 times, the big increase of the settling time from 1.0 to 10.08 times, but the overshoot is characterized with moderate increase from 0.45 to 0.96 times, in comparison with control system with PID controller tuned by the maximum stability degree method with iterations.

III. CONCLUSIONS

Analysing the obtained results for the case of tuning the PID controller to the model of object (1) by the maximum stability degree method with iteration it was found:

- By the rising the value of the stability degree J, it is raised the values of the tuning parameters and performance of the system as the rise time and the settling time is reduced, but overshoot is kept at the level $\sigma = 20 - 24\ \%$.

- With the increasing of the numerical value of the inertia T of the given model of object, for the same value of the maximum stability degree J, the performances of the system are equal (they do not depend on the numerical value of the inertia T).

- By the increasing the stability degree $J \rightarrow \infty$, it is raised the rise time t_c and settling time t_r is reduced.

- By the variation of the object parameters T with $\pm 50\%$ from the nominal value, the system is kept its operation mode but the settling time t_r is rising twice and overshoot is rising with 1.32 times, but at the reducing the T with 50% the settling time t_r is reduced by 31 % and overshoot σ is reduced by 40 %.

- At the action of the perturbation $p(t) = +1(t)$ the transient process becomes ideal, but at the action of perturbation $p(t) = -1(t)$ the settling time t_r rises by 1.1 times and overshoot σ by twice.

- It was found that at the values of the tuning parameters $k_p = 0$ and $k_i = 0$ the tuning is realized by the D component and by the variation of the $k_d = 2 \cdots 600$, the settling time varies $t_r = 59.87 \cdots 0.16$ s and transient process is aperiodic (for the case $T = 40, J = 3$).

- In the automatic control system with PID controller tuned by the genetic algorithm, with increasing the numerical value of the inertia of the object model, the numerical values of the system performances also increase.

- At the high values of the inertia T of the model of object, the highest performances are obtained for the case of tuning the PID controller by the maximum stability degree method with iterations.

ACKNOWLEDGMENT

This work was supported by the project 20.80009.5007.26 „Models, algorithms and technologies for the control, optimization and security of the Cyber-Physical systems".

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