Cinetostatic analysis of planetary precessional multiplier

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Abstract. The majority planetary precessional transmissions diagrams developed previously operate efficiently in reducer's regime. Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions that operate as multipliers. Multiple precessional gear theory, previously developed, did not take into consideration the influence of the diagram error of the linking mechanism in the processing device for gear wheel on the teeth profile. Functioning under the multiplication regime, the errors of the linking mechanism have major influence, which can lead to instant blocking of gear and to power losses. With this purpose, a thorough analysis was conducted on the motion development mechanism under multiplication, and on the teeth profile error generating source. This paper describes also the precessional transmission diagrams operate efficiently in multiplier's regime, and cinetostatic analysis of precessional multipliers.

1. Introduction

Depending on the structural diagram, precessional transmissions fall into two main types – *K-H-V* and *2K-H*, from which a wide range of constructive solutions with wide kinematical and functional options that operate in multiplier regime. The kinematical diagram of the precessional transmission *K-H-V* (fig. 1 [1]) comprises five basic elements: planet career *H*, satellite gear *g*, two central wheels *b* with the same number of teeth, controlling mechanism *W* and the body (frame). The roller rim of the satellite gear *g* gears internally with the sun wheels *b*, and their teeth generators cross in a point, so-called the centre of precession. The satellite gear *g* is mounted on the planet (wheel) career *H,* designed in the form of a sloped crank, which axis forms some angle with the central wheel axis θ.

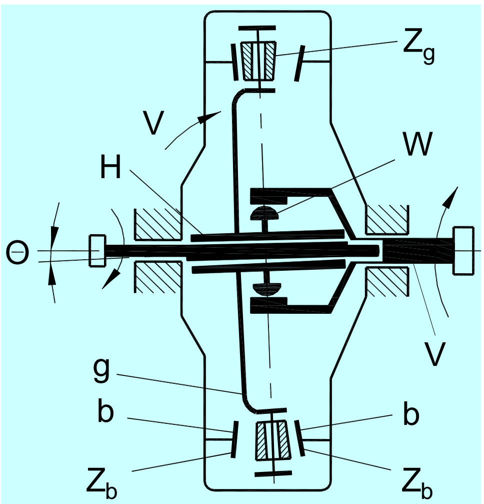

Figure 1. Conceptual diagrams of precesional transmissions that operates efficiently in the multiplication regime.

Revolving, the sloped crank *H* transmits sphero-spatial motion to the satellite wheel regarding the ball hinge installed in the centre of precession. For the transmission with the controlling mechanism designed as clutch coupling (fig.1.141, a), the gear ratio (gear reduction rate) varies in the limits:

$$i_{HV}^{g} = -\frac{z_g \cos\Theta - z_b}{z_b}; \; i_{HV}^{g} = -\frac{z_g \cos\Theta - z_b}{z_b \cos\Theta}, \quad (1)$$

reaching the extreme values of 4 times for each revolution of the crank ***H***. If necessary, this shortcoming can be eliminated using as a controlling mechanism the constant Cardan joint (Hooke's joint), the ball synchronous couplings, etc.

$$i_{HVmed}^{g} = -\frac{z_g - z_b}{z_b}. \quad (2)$$

For $z_g = z_b + 1$, $i_{HV}^{g} = -\frac{1}{z_b}$, the driving and driven shafts have opposite directions.

For $z_g = z_b - 1$, $i_{HV}^{g} = \frac{1}{z_b}$, the shafts revolve in the same direction.

This kinematical diagram of the precessional transmission ensures a range of gear ratios $i = 8...60$, but in the multiplication regime it operates efficiently only for the range of gear ratios $i = 8...25$. As well, in the controlling mechanism W, that operates with pitch angles of the semi couplings up to 3°, power losses occur reducing the efficiency of the multiplier on the whole.

To avoid power losses in the multiplier and to widen the kinematical options (fig. 2) [2] the conceptual diagram of the precessional multiplier with wide kinematical options was designed.

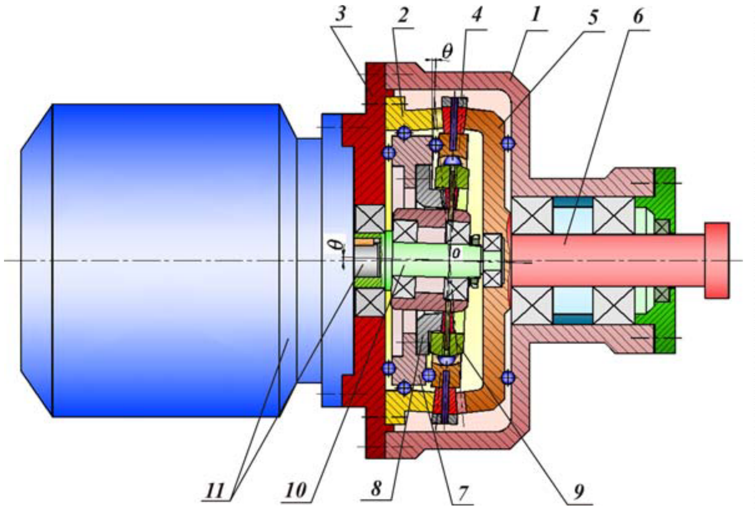

Figure 2. Planetary precessional multiplier with satellite gear mounted radially.

The planetary precessional multiplier comprises the following units: the housing 1, inside which the fixed sun wheel 2 is placed and connected rigidly to the housing cover 3, exterior satellite wheel 4 with the teeth in the shape of rollers, movable sun wheel 5, linked rigidly to the input shaft 6. The satellite wheel 3 is connected kinematically with the sloped flange of the disk 7, connected rigidly with the sun wheel 8, that gears with the interior satellite wheel 9 mounted unbound on the output crank shaft 10, and linked rigidly to the rotor generator 11. The exterior satellite wheel 4 is mounted void on gear bodies on exterior spherical surfaces of the interior wheel 9. The pitch angle of the output crank shaft 10 axis and of the sloped flange is equal to . The rotational motion of the input shaft 6 is transmitted to the movable sun wheel 5. Due to the difference in the number of teeth of wheel 5 and of the exterior satellite wheel (gear) 4 ($Z_6 = Z_5 \pm 1$), the last will have to carry out a precession motion around the fixed point O (centre of precession). Precessional motion around its axis is excluded as the number of teeth of the sun wheel 2 is equal to the number of rollers of the satellite gear 4 ($Z_2 = Z_4$). The precessional motion of the exterior satellite gear 4 is transformed, by means of an inclined flange of the disc 7, into rotational motion around the disc axis 7 that will revolve by the degree of multiplication

$$i_7 = -\frac{z_5}{z_4 - z_5}, \tag{3}$$

where Z_4 is the number of rollers of the exterior satellite gear 4;

Z_5 – is the number of teeth of the movable sun wheel 5.

The multiplied rotational motion of the disc 7 is transformed into multiplied precessional motion of the interior satellite gear 9, due to the difference in the number of teeth $Z_8 = Z_9 \pm 1$. At the rotation of disc 7 of the sun wheel 8 at an angle equal to the teeth angular pitch, the interior satellite gear will perform a complete precession cycle around point „O". The precessional motion of the interior satellite gear 9 is transformed by means of output crank shaft 10 into rotational motion, multiplied to the degree of multiplication

$$i_{10} = -\frac{z_8}{z_9 - z_8}, \tag{4}$$

where Z_8 is the number of teeth of the sun wheel 8,

Z_9 – is the number of rollers of the satellite gear 9.

The multiplied rotational motion of the output crank shaft 10 is transmitted to the generator rotor 11.

2. Analytic description of teeth profile and justification of precessional gear parameters selection

Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions that operate as multiplier. Multiple precessional gear theory, previously developed, did not take into consideration the influence of the diagram error of the linking mechanism in the processing device for gear wheel on the teeth profile.

Functioning under the multiplication regime, these errors have major influence, which can lead to instant blocking of gear and to power losses. With this purpose, a thorough analysis was conducted on the motion development mechanism under multiplication, and on the teeth profile error generating source. On the basis of fundamental theory of multiple precessional gear, previously developed, a new gear with modified teeth profile and the technology for its industrial manufacturing was proposed and patented [3].

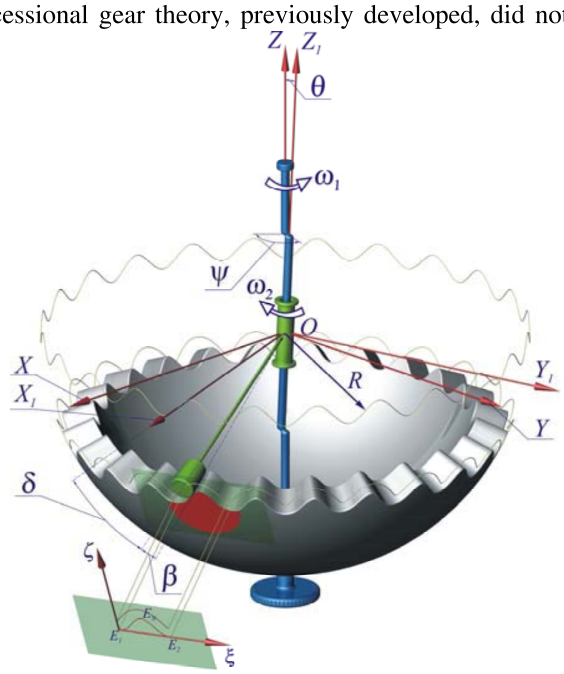

Figure 3. Tooth profile in normal section.

To ensure continuity of the transfer function and to improve the performances of precessional transmission under multiplication it is necessary to modify teeth profile with the diagram error value ψ_3 by communicating supplementary motion to the tool. In this case the momentary transmission ratio of the manufactured gear will be constant. Usually, in theoretical mechanics the position of the body making spherical-spatial motion is described by Euler angles.

The mobile coordinate system $OX_1Y_1Z_1$ is connected rigidly with the satellite wheel, which origin coincides with the centre of precession ***0*** (fig. 3) and performs spherical-spatial motion together with the satellite wheel relative to the motionless coordinate system ***OXYZ.***

The elaboration of the mathematic model of the modified teeth profile is based integrally on the mathematic model of teeth profile, previously developed by the authors. With this purpose it is necessary to present the detailed description of teeth profile without modification and, then, to present of the description of modified profile peculiarities.

3. Description of teeth profile designed on sphere

An arbitrary point *D* of the tool axis describes a trajectory relative to the fixed system according to the equations:

$$\begin{aligned}
X_D^m &= -\sin\delta \sin\left[Y_C^m \sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right];\\
Y_D^m &= -Y_C^m \cos\delta + Z_C^m \sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right];\\
Z_D^m &= -Y_C^m \sin\delta\left(\cos^2\psi + \cos\theta\sin^2\psi\right) - Z_C^m\cos\delta.
\end{aligned} \tag{5}$$

Index ***m*** means „*modified*".

The motion of point $\boldsymbol{D^m}$ compared to the movable system connected rigidly to the semi product is described by formulas:

$$\begin{aligned}
X_{1D}^m &= X_D^m\cos\frac{\psi}{Z_1} - Y_D^m\sin\frac{\psi}{Z_1};\\
Y_{1D}^m &= X_D^m\sin\frac{\psi}{Z_1} + Y_D^m\cos\frac{\psi}{Z_1};\\
Z_{1D}^m &= Z_D^m.
\end{aligned} \tag{6}$$

The projections of point $\boldsymbol{D^m}$ velocities is expressed by formulas:

$$
\begin{aligned}
&\dot{X}_D^m = -\sin\delta\cos\psi\left[Y_C^m\sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right]\dot{\psi} - \\
&-\sin\delta\sin\psi\left[\dot{Y}_C^m\sin\theta + \dot{Z}_C^m\left(1-\cos\theta\right)\cos\psi - Z_C^m\left(1-\cos\theta\right)\sin\psi\cdot\dot{\psi}\right]; \\
&\dot{Y}_D^m = -\dot{Y}_C^m\cos\delta + \dot{Z}_C^m\sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right] + \\
&+Z_C^m\sin\delta\left[-2\cos\psi\sin\psi + 2\cos\theta\sin\psi\cos\psi\right]\dot{\psi}; \\
&\dot{X}_{1D}^m = \dot{X}_D^m\cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}X_D^m\sin\frac{\psi}{Z_1} - \dot{Y}_D^m\sin\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}Y_D^m\cos\frac{\psi}{Z_1}; \\
&\dot{Y}_{1D}^m = \dot{X}_D^m\sin\frac{\psi}{Z_1} + \frac{\dot{\psi}}{Z_1}X_D^m\cos\frac{\psi}{Z_1} + \dot{Y}_D^m\cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}Y_D^m\sin\frac{\psi}{Z_1}.
\end{aligned}
\tag{7}
$$

The coordinates of point $\boldsymbol{E^m}$ on the sphere is calculated by formulas:

$$
\begin{aligned}
&X_{1E}^m = k_2^m Z_{1E}^m + d_2^m; \\
&Y_{1E}^m = k_1^m Z_{1E}^m - d_1^m; \\
&Z_{1E}^m = \frac{(k_1^m d_1^m - k_2^m d_2^m) - \sqrt{(k_1^m d_1^m - k_2^m d_2^m)^2 + (k_1^{m2} + k_2^{m2} + 1)\cdot(R_D^2 - d_1^{m2} - d_2^{m2})}}{k_1^{m2} + k_2^{m2} + 1},
\end{aligned}
\tag{8}
$$

where:

$$
k_1^m = \frac{X_{1D}^m\left(X_{1D}^m\dot{X}_{1D}^m + Y_{1D}^m\dot{Y}_{1D}^m\right) + Z_{1D}^{m\,2}\dot{X}_{1D}^m}{Z_{1D}^m\left(X_{1D}^m\dot{Y}_{1D}^m - Y_{1D}^m\dot{X}_{1D}^m\right)}; \quad k_2^m = -\frac{\left(k_1^m Y_{1D}^m + Z_{1D}^m\right)}{X_{1D}^m};
$$

$$
d_1^m = \frac{R_D^2\cos\beta\,\dot{X}_{1D}^m}{\left(X_{1D}^m\dot{Y}_{1D}^m - \dot{X}_{1D}^m Y_{1D}^m\right)}; \quad d_2^m = \frac{\left(R_D^2\cos\beta + d_1^m Y_{1D}^m\right)}{X_{1D}^m}.
$$

According to the obtained analytical relations a soft for the calculation and generation of teeth was developed in CATIA V5R7 modelling system that allowed obtaining the modified trajectories of points $E^m{}_e$ and $E^m{}_i$ on the spherical front surfaces, both exterior and interior ones, by which the teeth surface was generated (fig. 4).

Projection of point $\boldsymbol{E^m}$ on the tooth transversal plane has the following coordinates:

$$
X_E^{\prime\prime m} = \varepsilon^m\cdot X_{1E}^m, \quad Y_E^{\prime\prime m} = \varepsilon^m\cdot Y_{1E}^m, \quad Z_E^{\prime\prime m} = \varepsilon^m\cdot Z_{1E}^m, \tag{9}
$$

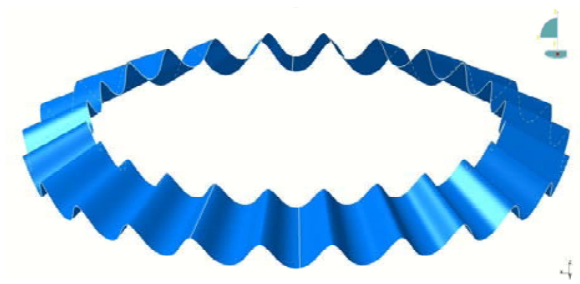

Figure 4. Teeth generating surface.

where $\varepsilon^m = -\dfrac{D}{AX_{1E}^m + BY_{1E}^m + CZ_{1E}^m}$.

The modified teeth profile in plane is described by the equations:

$$
\begin{aligned}
&\xi^m = X_E^{\prime\prime m}\cos\frac{\pi}{Z_1} + \left[R_D\cos\left(\delta+\theta+\beta\right) + Y_E^{\prime\prime m}\right]\sin\frac{\pi}{Z_1}; \\
&\zeta^m = X_E^{\prime\prime m}\sin\gamma\sin\frac{\pi}{Z_1} - \left[R_D\cos\left(\delta+\theta+\beta\right) + Y_E^{\prime\prime m}\right]\sin\gamma\cos\frac{\pi}{Z_1} + \\
&+\left[R_D\sin\left(\delta+\theta+\beta\right) + Z_E^{\prime\prime m}\right]\cos\gamma.
\end{aligned}
\tag{10}
$$

A wide range of modified teeth profiles with different geometrical parameters were generated in MathCAD 2010 Professional software (fig. 5). The solid model of a gear wheel is shown in fig. 6.

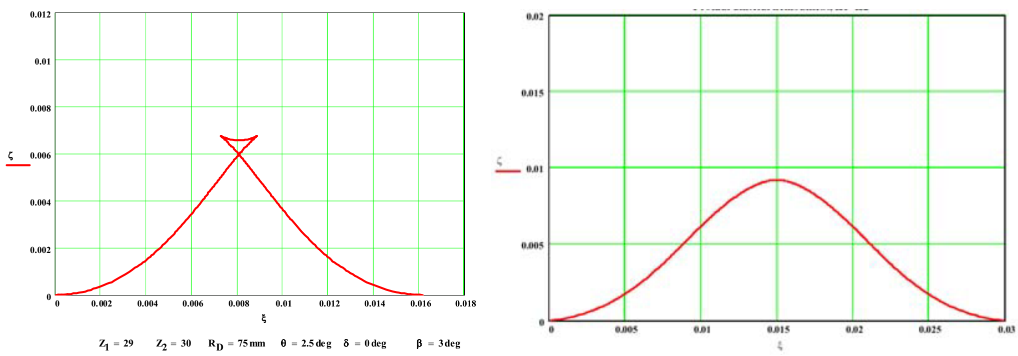

Figure 5. Teeth profiles for multipliers.

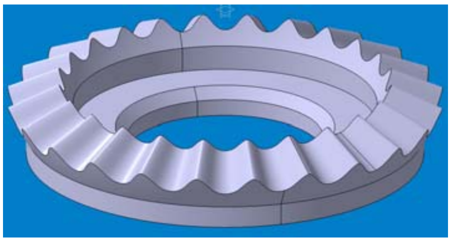

Figure 6. Computerised model of the sun gear.

Based on the carried out research it was established that from the point of view of decreasing energy losses in gearing, in the multiplication mode of operation, the gearing angle should be $\alpha > 45^0$, and the nutation angle (the pitch angle of the crank shaft) should be – $\theta \leq 2{,}5^0$. This is dictated by the reverse principle of movement in the multipliers compared to the reducers: the axial component of the normal force in gear must be maximal to drive the crank shaft in the rotation movement through the satellite wheel.

References

[1] Bostan I, Dulgheru V, Sochirean A., Babaian I. Antologia invențiilor, Vol. 1. Transmisii planetare precesionale. Chișinău (Bons Offices), 2011. 594p. ISBN 978-9975-4100-9-0.
[2] Bostan I., Dulgheru V., Ciobanu R., Ciobanu O. Patent No. 3153 B1 (MD), CIB F16H1/32. Planetary precessional multiplier. Publ. BOPI - 2006. - No. 9.
[3] Bostan I., Țopa M., Dulgheru V., Vaculenco M. Patent No. 1886 B1 (MD), CIB F16H1/32; B23F9/06. Precessional gearing and procedure for its fulfilment / T.U.M. – No.2000 – 071. Publ. BOPI – 2002. – No.3.

Acknowledgments

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