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EFFICIENCY INDICES OF INVESTMENT IN IT PROJECTS WITH EQUAL LIVES

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Abstract. Theoretical results not always give an unambiguous answer regarding the preference of using the indices of efficiency of investment in IT projects with equal lives. To complement some of such results, the Net Present Value (NPV), Profitability (PI) and Internal Rate of Return (IRR) indices are researched by computer simulation. In this aim, a model of comparative analysis of projects with equal lives is defined and the SIMINV application is made up. Using SIMINV, the percentage of cases when the solutions, obtained according to indices of each of the pairs {NPV, PI}, {NPV, IRR}, {PI, IRR} or of the triplet {NPV, PI, IRR}, differ for seven groups of alternatives of initial data is determined. Based on done calculations, some properties of indices were identified, including: the quantitative features and the character of dependences on initial data; the average percentage of cases with different solutions, which is of approx. 9 % for the pair of indices PI and IRR, and of 34-35 % for the other two pairs of indices specified above. On average, the solutions of comparing the efficiency of projects with equal lives, obtained using the NPV, PI and IRR indices, does not coincide in more than 1/3 of cases.

Keywords: *comparative analysis, computer simulation, internal rate of return, net present value, profitability index.*

Rezumat. Rezultatele teoretice nu întotdeauna oferă un răspuns univoc privind preferințele de aplicare a indicilor de eficiență a investițiilor în proiecte IT de durată similară. Pentru a complementa rezultate cunoscute, indicii Valoarea Actualizată Netă (VAN), Profitabilitatea (PI) și Rata Internă de Rentabilitate (RIR) sunt cercetați prin simulare informatică. În acest scop este definit un model de analiză comparativă a proiectelor de investiții de aceeași durată și este alcătuită aplicația informatică SIMINV. Folosind SIMINV, este determinat procentajul cazurilor, în care soluțiile, obținute folosind indicii fiecăreia dintre perechile {NPV, PI}, {NPV, IRR} și {PI, IRR} sau cei ai tripletului {NPV, PI, IRR}, diferă pentru șapte grupuri de alternative de date inițiale. Pe baza calculelor efectuate au fost identificate unele proprietăți ale indicilor, inclusiv: caracteristicile cantitative și caracterul unor dependențe de datele inițiale; procentajul mediu al cazurilor cu soluții diferite, care este de cca. 9 % pentru perechea de indici PI si IRR si de 34-35 % pentru celelalte doua perechi de indici specificați mai sus. În

medie, soluțiile de comparare a eficienței proiectelor de aceeași durată, obținute folosind indicii VAN, PI și RIR, nu coincid în peste 1/3 din cazuri.

Cuvinte cheie: *analiză comparativă, simulare informatică, rata internă de rentabilitate, valoare actualizată netă, indice de profitabilitate.*

1. Introduction

As is well known, offered advantages impose the computerization of diverse activities implying respective investments. A decision of investment in an IT project is usually made on the basis of efficiency criteria/indices.

In economic analysis of IT projects (i-projects), the reasonable choice of indices to estimate the solution alternatives is of prime importance. For the assessment of economic efficiency of investment projects, such indicators are recommended as: profit, profit rate [1-3], payback period on investment, net present value [1, 4-6], profitability index [1, 5, 7], internal rate of return [1, 2, 7], return on investment [1, 8], economic return on investments [3, 8], adjusted expenditure [8], total costs of ownership [9] and so on.

Depending on project product and its field of use, the set of applied indices may differ. In a specific project, a small set of indices is usually applied. It is recommended to analyze 7 ± 2 indices [7]. Typically, 1-3 core indices and a few auxiliary indices are used. According to [4], the NPV, IRR, and discounted payback period (DPP) indices are most often recommended to be used. Along with the NPV, PI, IRR, and DPP ones, in [10] the Finite Value of the project and Modified Internal Rate of Return indices are explored; for a concrete project, using all these five indices leads to the same decision – it is appropriate to invest. But, of course, there may be many cases where the results differ. How often such situations occur? Known theoretical results do not give an unambiguous answer to this question. At the same time, to identify them computer simulation can be used.

Monte-Carlo method is largely used to assess financial risks in investment projects. For example, risk assessment for environmental projects using this method is provided in [11]. To select a project for the research, characteristics of 63 projects in the field were analyzed. By computer simulation it was determined the cumulated probability that the project value and execution period will be higher than the initially estimated values. A Monte-Carlo approach to assess financial risk in investment projects is used also in [12]. As a result, a new contribution to the field is made: the proposed risk scale offers five classifications regarding the degree of loss. In [13], a multiple criteria procedure based on stochastic dominance and PROMETEE II methodology is proposed. The first step of this procedure is computer simulation and the uncertainty of processes is taken into account by special stochastic dominance rule. There are many other aspects regarding the selection of investment projects which are explored by computer simulation.

In order to extend the theoretical results regarding the estimation of efficiency of investment in i-projects with equal lives, in this paper the net present value, profitability and internal rate of return indices are researched comparatively by computer simulation mainly to identify the frequency of non-coincidence of the obtained solutions.

2. Materials, Methodology and Methods

The comparative analysis of 16 indices of economic efficiency of investment in i-projects, performed in [14, 15] and based on correlation between indices, the specificity of the time value of money, the different duration of projects and also the range and importance

of the characterized aspects, show that as basic indices, for projects the revenues from the implementation of which can be estimated with reasonable efforts, it is opportune to use three: NPV, IRR and PI, eventually in conjunction with the equivalent annual value method. The last method allows the appropriate comparison of projects with different lifetimes that is not the case of this paper.

Below, **the approach** defined in [16] is followed, but with adaptations for projects of equal lives. Let I are investments and CF_t are cash flows in year t related to the project. Then NPV, IRR and PI indices are determined as [1, 7]:

$$\text{NPV} = \sum_1^D \frac{\text{CF}_t}{(1+d)^t} - I^C, \quad \sum_1^D \frac{\text{CF}_t}{(1+\text{IRR})^t} - I^C = 0, \quad PI = 1 + \frac{NPV}{I^C} \qquad (1)\text{-}(3)$$

where d is the discount rate.

These three indices form a Pareto set: no one of the three can always replace the use of one or two of the other indices, in sense of obtaining the same solutions when comparing projects. At the same time, there are particular cases when the use of all or two of the three indices for comparing two projects leads to the same solution. It is of interest how frequently such cases take place. To this and some other aspects, the answer can be obtained by computer simulation.

Let's compare two i-projects, 1 and 2, with equal lifetimes $D_1 = D_2 = D$ the revenues from the implementation of which can be estimated with reasonable efforts. When updating the values of indices, as time reference point will be used the time of projects launch in operation; this time is the same for both projects. It is required to identify, by computer simulation, the percentages of cases when the solutions, obtained using indices of each of the pairs {NPV, PI} (NP) – q_{NP}, {NPV, IRR} (NR) – q_{NR}, {PI, IRR} (PR) – q_{PR} and also of at least one of these three pairs – q_{NPR}, leads to different solutions. Obviously, the percentage of coincidence of all solutions when applying the three indices (NPV, PI and IRR) is equal to 100 – q_{NPR}.

The discount rate d will be considered constant and equal for the two projects, but the values of CF_t and also those of I can be different for the two projects. They are also introduced two parameters, g and v. Parameter g value is determined for reasons of ensuring a given value r for the IRR index. So, from Eq.(2) at $CF_t = CF$, $t = 1, 2, ..., D$, one has

$$\sum_{t=1}^D \frac{\text{CF}_t}{(1+r)^t} - I = \text{CF} \sum_{t=1}^D \frac{1}{(1+r)^t} - I = \text{CF} \frac{1 - (1+r)^{-D}}{r} - I = \text{CF}/g - I = 0 \ ,$$

that is

$$g = \text{CF}/I = r/[1 - (1+r)^{-D}]. \qquad (4)$$

Thus, g depends on r and D and, at the same time, it establishes the relation between the value I of investment and the average value CF of cash flows CF_t, $t = 1, 2, ..., D$. Of course, at $CF_t \neq CF$, $t = 1, 2, ..., D$ the IRR value isn't equal to r, but it is relatively close to it.

In its turn, parameter v characterizes the range of relative variation of CF_t with respect to CF. Therefore, the value of v is assigned according to the value CF = gI, namely

$$v = (\text{CF} - \text{CF}_{min})/\text{CF} = (\text{CF}_{max} - \text{CF})/\text{CF}. \qquad (5)$$

So,

$$CF_{min} = CF(1 − v) = gI(1 − v),\tag{6}$$

$$CF_{max} = CF(1 + v) = gI(1 + v)\tag{7}$$

and

$$CF_t \in [CF_{min}; CF_{max}], t = 1, 2, ..., D.\tag{8}$$

In calculations, for parameters d, r, v, D and I will be used values from the ranges argued and used in [16], namely: $d \in [0.05; 0.14]$, $r \in [0.1; 0.9]$, $v \in [0.1; 0.5]$, $D \in [1; 10]$ and $I \in [100; 1000]$. Using these ranges of values, a very large number of alternatives of initial data can be formed. From these, as in [16], seven groups of alternatives, a1-a7, are selected. In all of them, the CF_t values are generated randomly at uniform repartition in the respective range as follows (taking into account Eq.(6)-Eq.(8)):

$CF_{1t} \in [CF_{1min}; CF_{1max}]$, where $CF_{1min} = g(1 − v)I_1$ and $CF_{1max} = g(1 + v)I_1$;
$CF_{2t} \in [CF_{2min}; CF_{2max}]$, where $CF_{2min} = g(1 − v)I_2$ and $CF_{2max} = g(1 + v)I_2$.

In alternative a6, the values of I and D are also generated randomly at uniform repartition in the respective range: $I_1 \in [100; 1000]$, $I_2 \in [100; 1000]$ and $D_1 = D_2 \in [1; 10]$. Additionally, in alternative a7 the values of r and v are generated randomly in the respective range: $r \in [0.1; 0.9]$ and $v \in [0.1; 0.9]$. At the same time, any such generated set of initial data is accepted only if $NPV_1 > 0$, $NPV_2 > 0$ and $|IRR_1 − IRR_2| \geqslant \varepsilon$. The reason of using the parameter ε ($\varepsilon = 0.005$) is to take into account the error of calculations when determining the IRR_1 and IRR_2 values.

Thus the seven groups of alternatives are:

a1) the reference group (dependence on d): $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = 500$; $r = 0.2$; $v = 0.5$;

a2) dependence on D: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = \{1, 2, 3, ..., 10\}$; $I_1 = 1000$, $I_2 = 500$; $r = 0.2$; $v = 0.5$;

a3) dependence on I_2: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = \{100, 200, 300, ..., 900, 1000\}$; $r = 0.2$; $v = 0.5$;

a4) dependence on r: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = 500$; $r = \{0.1, 0.2, 0.3, ..., 0.9\}$; $v = 0.5$;

a5) dependence on v: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = 500$; $r = 0.2$; $v = \{0.1, 0.2, 0.3, ..., 0.9\}$;

a6) dependence on $d+$ (on d when D_2 and I_2 are generated randomly – partial general group): $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 \in [1; 10]$; $I_1 \in [100; 1000]$, $I_2 \in [100; 1000]$; $r = 0.2$; $v = 0.5$;

a7) dependence on $d·$ (the general group): $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 \in [1; 10]$; $I_1 \in [100; 1000]$, $I_2 \in [100; 1000]$; $r \in [0.1; 1.0]$; $v \in [0.1; 0.9]$.

For each of the seven alternatives, the respective percentages q_{NP}, q_{NR}, q_{PR}, q_{NPR} and f have to be determined. Here f is the dependence on respective parameter (parameters) of the percentage of generated sets of initial data for which at least one of the following requirements take place: $NPV_1 < 0$, $NPV_2 < 0$ or $|IRR_1 − IRR_2| > \varepsilon$ (percentage of failure cases).

The algorithm, for the determination of percentages $q_{NP}(d)$, $q_{NR}(d)$, $q_{PR}(d)$, $q_{NPR}(d)$ and $f(d)$ in general case – group a7, is the following.

1. Initial data: d = {0.05, 0.06, 0.07, …, 0.14}; $D_1 = D_2 \in [D_{min}; D_{max}]$; $I_1 \in [I_{min}; I_{max}]$, $I_2 \in [I_{min}; I_{max}]$; $r \in [r_{min}; r_{max}]$; $v \in [v_{min}; v_{max}]$, N (total number of values for d), K (total number of initial data values for the done value of d - sample size). $n := 1$, $d := d_o$.

2. $m_f := 0$, $m_{NP} := 0$, $m_{NR} := 0$, $m_{PR} := 0$, $m_{NPR} := 0$ and $k := 1$.

3. Generation, at uniform random distribution, of the values of quantities $D_1 = D_2 = D \in [D_{min}; D_{max}]$; $I_1 \in [I_{min}; I_{max}]$, $I_2 \in [I_{min}; I_{max}]$ and $g := r/[1 - (1 + r)^{-D}]$.

4. $CF_{1min} := g(1 - v)I_1$, $CF_{1max} := g(1 + v)I_1$, $CF_{2min} := g(1 - v)I_2$, $CF_{2max} := g(1 + v)I_2$ and generation, at uniform random distribution, of the values of quantities $CF_{1t} \in [CF_{1min}; CF_{1max}]$, t = 1, 2, …, D and $CF_{2t} \in [CF_{2min}; CF_{2max}]$, t = 1, 2, …, D.

5. Determination of NPV_1 according to Eq.(1). If $NPV_1 < 0$, then $m_f := m_f + 1$ and go to Step 10.

6. Determination of NPV_2 according to Eq.(1). If $NPV_2 < 0$, then $m_f := m_f + 1$ and go to Step 10.

7. Determination of IRR_1 and IRR_2 taking into account the Eq.(2). If $|IRR_1 - IRR_2| \leqslant \varepsilon$, then $m_f := m_f + 1$ and go to Step 10.

8. Determination of PI_1 and PI_2 according to Eq.(3).

9. Identification and counting the numbers m_{NP}, m_{NR}, m_{PR} and m_{NPR} of cases when the solutions, obtained using indices of each of the pairs NP, NR and PR, and, respectively, at least of one of these pairs, leads to different solutions.

10. If $k < K$, then $k := k + 1$ and go to Step 3.

11. $q_{NP}(d) := 100m_{NP}/(K - m_f)$, $q_{NR}(d) := 100m_{NR}/(K - m_f)$, $q_{PR}(d) := 100m_{PR}/(K - m_f)$, $q_{NPR}(d) := 100m_{NPE}/(K - m_f)$ and $f(d) := 100m_f/K$.

12. If $n < N$, then $d := d + \Delta d$ and go to Step 2.

13. Taking over the simulation results. Stop.

Similar, with respective adaptations, are the algorithms for the groups of alternatives a1-a6. To implement the seven algorithms, the computer application SIMINV in C++ was made up.

3. Results and Discussion

To achieve the goal defined in Section 1, respective calculations were performed using the computer application SIMINV. Some of the obtained results are systemized in this section. Each set of initial data characterizes two concrete projects, 1 and 2. According to the algorithm and the seven groups of alternatives described in Section 2, a sample of 100000 was generated. So, were generated for the group of alternatives:

a1, a6 and a7 by 10×10^5 = 1 mil sets of initial data;
a2 and a3 by $10 \times 10 \times 10^5$ = 10 mil sets of initial data;
a4 and a5 by $10 \times 9 \times 10^5$ = 9 mil sets of initial data.

3.1. The number of initial data generation failures

The approach, used to establish and generate initial data sets, doesn't ensure the requirements of $NPV_1 > 0$ and $NPV_2 > 0$. Also there exists an error when calculating the IRR_1 and IRR_2 values using the dichotomy method whithin the algorithm described in Section 2. That is why the algorithm counters the total number of cases of failure m_f (if takes place at least one of the inequalities: $NPV_1 < 0$, $NPV_2 < 0$ or $|IRR_1 - IRR_2| > \varepsilon$). This number is used when calculating the values of percentages $q_{NP}(\cdot)$, $q_{NR}(\cdot)$, $q_{PR}(\cdot)$, $q_{NPR}(\cdot)$ and $f(\cdot)$. If this number is too large, then the calculation errors of obtained percentages are also significant. Therefore it is important to know its value.

In Figure 1, the dependences of f on d for the groups of alternatives of initial data a1, a6 and a7 are shown. The character of these dependencies is largely similar to those for the case of unequal lives described in [16], however the absolute value is higher, but not exceeding 48.2 %.
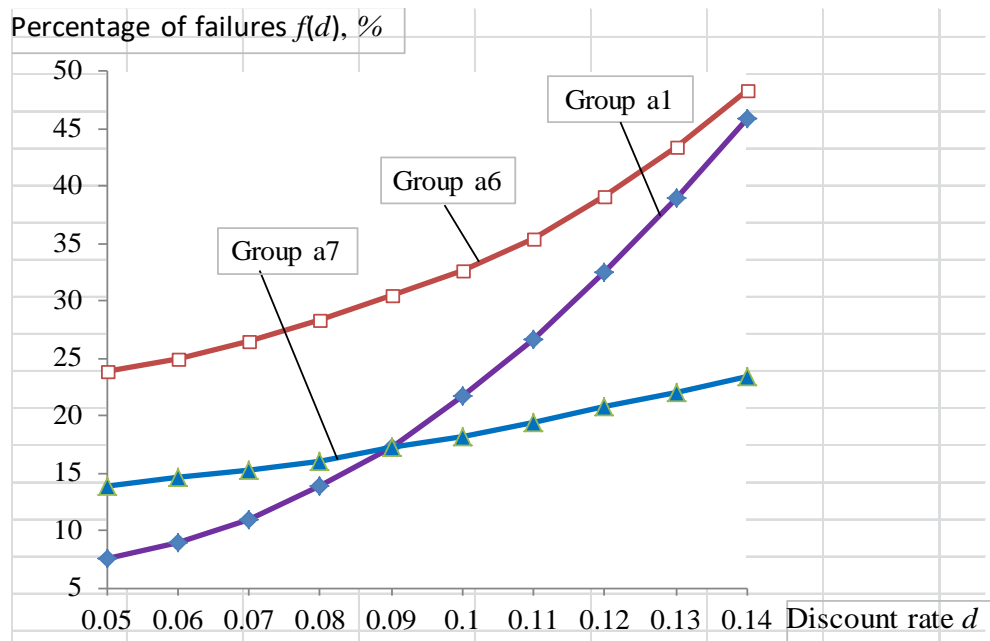

Figure 1. Percentages of failures when generating the sets of initial data.

The results of performed calculations show also that for the group of alternatives of initial data:

    a2 the dependence $f(d,D)$ is increasing on d, but is decreasing on D, the range of values being [7.4; 69.3] % at $d = 0.08$ and overall [6.0; 74.3]%;

    a3 the dependence $f(d,l_2)$ is increasing on d and is very little dependent on l_2, the range of values being [13.5; 14.0]% at $d = 0.08$ and overall [7.3; 46.1]%;

    a4 the dependence $f(d,r)$ is increasing on d, but is decreasing on r, the range of values being [1.6; 71.9] % at $d = 0.08$ and overall [1.5; 97.7] %, but [1.5; 45.9] % at $r \geqslant 0.2$;

    a5 the dependence $f(d,v)$ is increasing on d; with refer to v, it initially is decreasing and after is increasing, the range of values being [8.2; 40.1] % at $d = 0.08$ and overall [5.8; 63.6]%.

So, for all seven groups a1-a7 of alternatives of initial data the dependences $f(\cdot)$ are increasing on d, the overall range of values being of [1.5; 74.3] %, except the case of group a4 at $r = 0.1$ when the high limit is of 97.7%. Thus, in case of group a4 at $r = 0.1$, the sample of initial data is of 100000(100 − 97.7)/100 = 2300 alternatives and usually is sufficient. In all other cases, the sample of initial data exceeds 100000(100 − 74.3)/100 = 25700 alternatives and is good.

3.2. Frequency of cases for which the obtained solutions differ

Computer simulation using SIMINV was performed for all seven groups of alternatives defined in Section 2. Some results are described below.

The group of alternatives a1 - dependence on d. Initial data: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $l_1 = 1000$, $l_2 = 500$; $r = 0.2$; $v = 0.5$. The obtained dependences $q_{NP}(d)$, $q_{NR}(d)$, $q_{PR}(d)$ and $q_{NPR}(d)$ are shown in Figure 2.

Figure 2 shows that all mentioned dependences are decreasing on d. At the same time, dependences $q_{NP}(d)$ and $q_{NR}(d)$ practically coincide, and dependence $q_{NPR}(d)$ is close to the first two. Also, one has: $q_{NPR}(d) > q_{NP}(d) \approx q_{NR}(d) \gg q_{PR}(d)$. The obtained ranges of values for the four dependences are specified in Table 1.
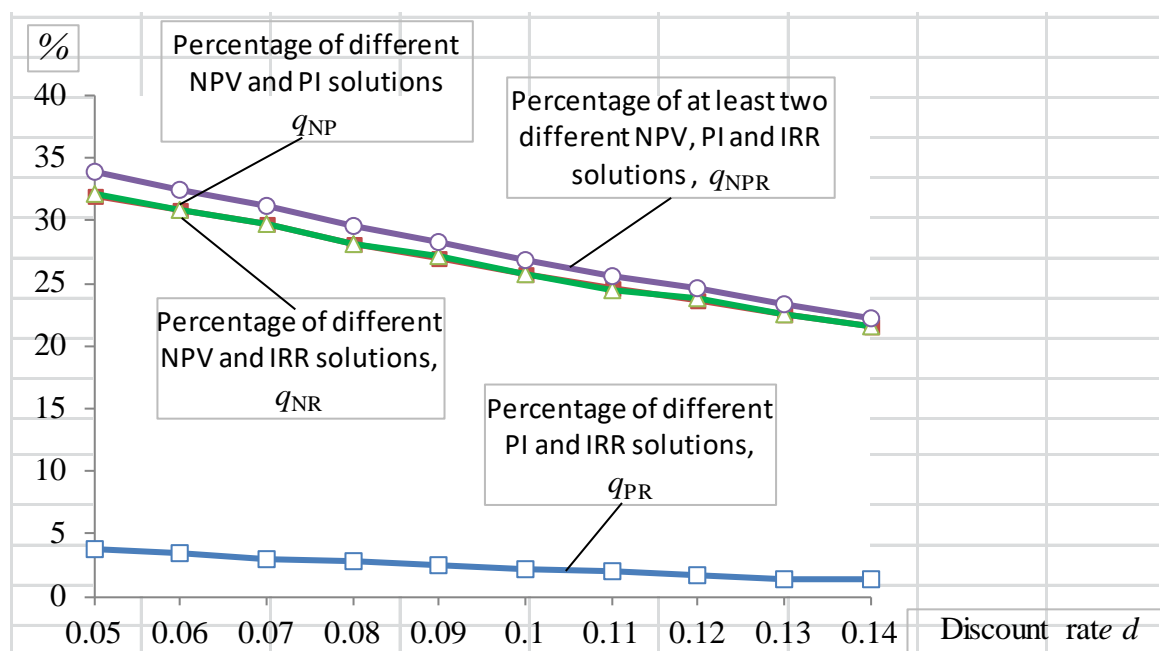

Figure 2. Percentages $q_{NP}(d)$, $q_{NR}(d)$, $q_{PR}(d)$ and $q_{NPR}(d)$.

Table 1

The ranges of values for the four dependences on discount rate $d \in [0.05; 0.14]$, %

Indicators	$q_{NP}(d)^1$	$q_{NR}(d)^2$	$q_{PR}(d)^3$	$q_{NPR}(d)^4$
Minimum of $q(d)$	21.6	21.6	1.30	22.2
Maximum of $q(d)$	32.1	32.1	3.84	34.0

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI, and IRR solutions.

Based on data of Table 1, it can be concluded that, on average, exists a considerable number of cases ($q_{NPR}(d) \in [22.2; 34.0]$%) when the use of at least two of the three examined indices (NPV, PI and IRR) leads to different solutions. The use of pairs of compared indices NP and NR also can lead to different solutions in a significant number of cases the respective range of values being approx. the same and equal to [21.6; 32.1]%. The narrowest range (the difference between the high and low limits) is that of $q_{PR}(d)$ equal to 3.84 – 1.30 = 2.54 %. Also, because of the smallest values of percentages $q_{PR}(d) \in [1.30; 3.84]$%, from the three compared indices, the PI and IRR are the closest to each other.

The group of alternatives a2 - dependence on D. Initial data: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = D = \{1, 2, 3, ..., 10\}$; $I_1 = 1000$, $I_2 = 500$; $r = 0.2$; $v = 0.5$. In graphical form, the dependences $q_{NP}(D_2)$, $q_{NR}(D)$, $q_{PR}(D)$ and $q_{NPR}(D)$ at $d = 0.08$ are presented in Figure 3. One can see that the character of the three dependences on D are different: that of $q_{PR}(D)$ is slowly increasing; those of $q_{NP}(D)$, $q_{NR}(D)$ and $q_{NPR}(D)$ are decreasing at $D \leqslant 2$ and are increasing at $D_2 > 2$. Also, as in Figure 2, dependences $q_{NP}(d)$, $q_{NR}(d)$ practically coincide, and dependence $q_{NPR}(d)$ is close to the first two. At the same time, one has: $q_{NPR}(d) > q_{NP}(d) \approx q_{NR}(d) \gg q_{PR}(d)$.

The obtained ranges of values for the four dependences are specified in Table 2.

The ranges of values for the four dependences on lifetime *D* at disc. rate *d* ∈ [0.05; 0.14], %

Indicators	$q_{NP}(D)$[1]	$q_{NR}(D)$[2]	$q_{PR}(D)$[3]	$q_{NPR}(D)$[4]
Minimum of $q(D)$	20.3	21.1	0	21.1
Maximum of $q(D)$	47.7	47.6	7.1	51.2

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI, and IRR solutions.
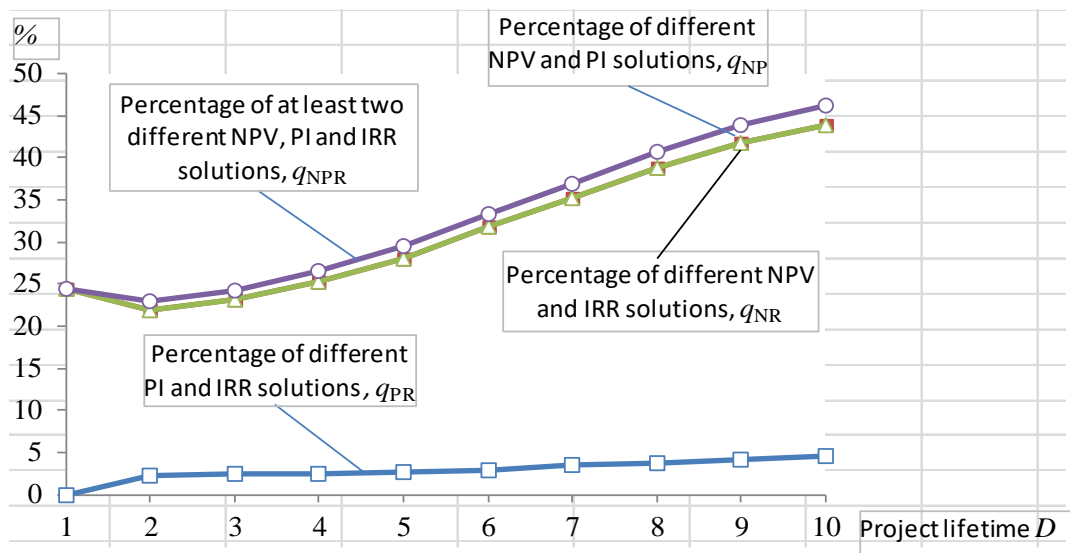

Figure 3. Percentages $q_{NP}(D)$, $q_{NR}(D)$, $q_{PR}(D)$ and $q_{NPR}(D)$.

It can be seen that there can be a large number of cases when the use of at least two of the three examined indices leads to different solutions ($q_{NPR}(D) \in [21.1; 51.2]$%). The use of pairs of compared indices NP and NR also can lead to different solutions in a significant number of cases the respective range of values being approx. the same, but not exceeding 47.8%. The narrowest range is that of $q_{PR}(D)$ equal to 7.1 %. Also, because of the smallest values of percentage $q_{PR}(D) \in [0; 7.1]$%, from the three compared indices, the PI and IRR are the closest to each other. At $D = 1$, the solutions obtained using these two indices coincide ($q_{PR}(D=1) = 0$) no matter of the $d \in [0.05; 0.14]$ value.

The group of alternatives a3 - dependence on I_2. Initial data: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = \{100, 200, 300, ..., 900, 1000\}$; $r = 0.2$; $v = 0.5$. Some results of calculations with refer to dependences $q_{NP}(I_2)$, $q_{NR}(I_2)$, $q_{PR}(I_2)$ and $q_{NPR}(I_2)$ at $d = 0.08$ are shown in Figure 4.

From Figure 4 one can see that percentages $q_{PR}(I_2)$ practically does not depend on I_2, while the other three dependences are decreasing on I_2, being very close to each other. Moreover, the dependences $q_{NP}(I_2)$ and $q_{NR}(I_2)$ practically coincide, except the case of $I_1 = I_2 = 1000$, when $q_{NP}(I_2) = 0$ no matter of the $d \in [0.05; 0.14]$ value. So, at $I_1 = I_2 = 1000$, the solutions obtained using the NPV and PI indices coincide no matter of the $d \in [0.05; 0.14]$ value. This fact is obvious if to take into account Eq.(1) and Eq.(3). The obtained ranges of values for the four dependences at $d \in [0.05; 0.14]$ are systemized in Table 3.
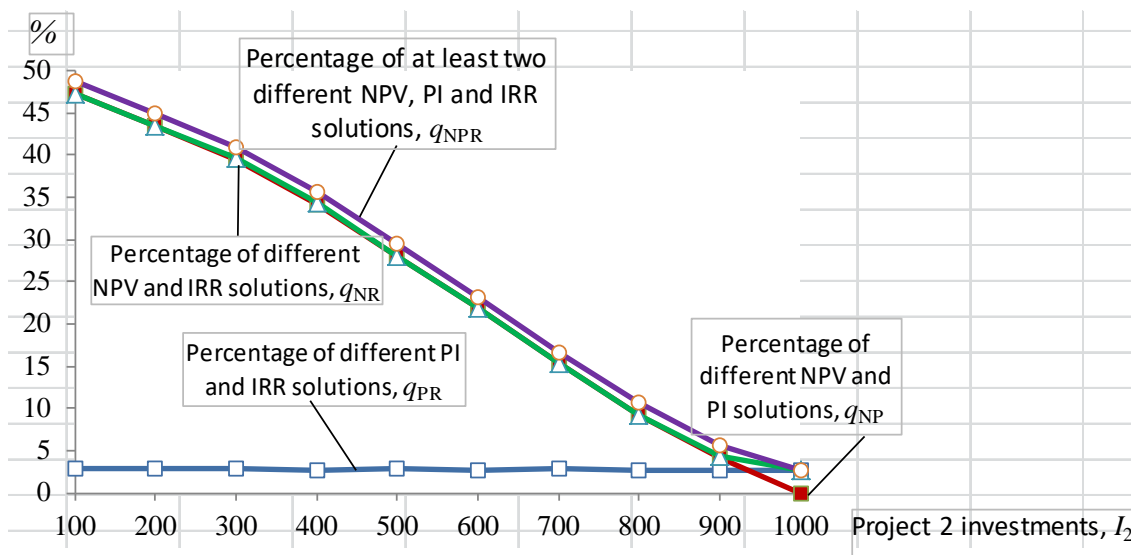
Figure 4. Percentages $q_{NP}(I_2)$, $q_{NR}(I_2)$, $q_{PR}(I_2)$ and $q_{NPR}(I_2)$.

The ranges of values for the four dependences on investments I_2 at rate $d \in [0.05; 0.14]$, %

Indicators	$q_{NP}(I_2)$[1]	$q_{NR}(I_2)$[2]	$q_{PR}(I_2)$[3]	$q_{NPR}(I_2)$[4]
Minimum of $q(I_2)$	0	1.3	1.2	1.3
Maximum of $q(I_2)$	48.3	48.3	3.9	50.3

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI, and IRR solutions.

As in previous two groups of alternatives, there can be a considerable number of cases when the use of at least two of the three examined indices leads to different solutions ($q_{NPR}(d) \in [1.3; 50.3]$ %). The use indices NP and NR also can lead to different solutions in a significant number of cases, but not exceeding 48.3 %. At the same time, at $I_1 = I_2 = 1000$ the solutions obtained when using indices NP and NR coincide ($q_{NP}(I_2) = 0$) no matter of the $d \in [0.05; 0.14]$ value. The narrowest range is that of $q_{PR}(I_2)$ equal to 3.9 − 1.2 = 2.7 %. Also, because of the smallest values of percentage $q_{PR}(d) \in [1.2; 3.9]$ %, from the three compared indices, the PI and IRR are usually the closest to each other.

The group of alternatives a4 - dependence on r. Initial data: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = 500$; $r = \{0.1, 0.2, 0.3, ..., 0.9\}$; $v = 0.5$. The obtained dependences $q_{NP}(r)$, $q_{NR}(r)$, $q_{PR}(r)$ and $q_{NPR}(r)$ at $d = 0.08$ are shown in Figure 5.

One can see that all four examined dependencies are increasing on r and those of $q_{NP}(r)$ and $q_{NR}(r)$ practically coinciding with each other ($q_{NP}(r) \approx q_{NR}(r)$). It is also increasing on r the discrepancy between $q_{NP}(r) \approx q_{NR}(r)$ and $q_{NPR}(r)$. Compared to the previous three groups of alternatives, the increase on r of $q_{PR}(r)$ is stronger. At the same time, take place $q_{NR}(r) < q_{NP}(r) \approx q_{NPE}(r) < q_{NPR}(r)$ and $q_{PR}(r) = 0$ at $\{r = 0.1, d = 0.14\}$. The obtained ranges of values, for the four dependences at $d \in [0.05; 0.14]$, are systemized in Table 4.

As in previous three groups of alternatives, there can be a considerable number of cases when the use of any two of the three examined indices leads to different solutions. The largest range of values is that of $q_{NPR}(d)$ equal to 58.7 − 13.4 = 45.3 %, and the narrowest one is that of $q_{PR}(d)$ equal to 19.1 %. Also, because of the smallest values of percentages $q_{PR}(d) \in [0; 19.1]$%, from the three compared indices the PI and IRR are the closest to each other.
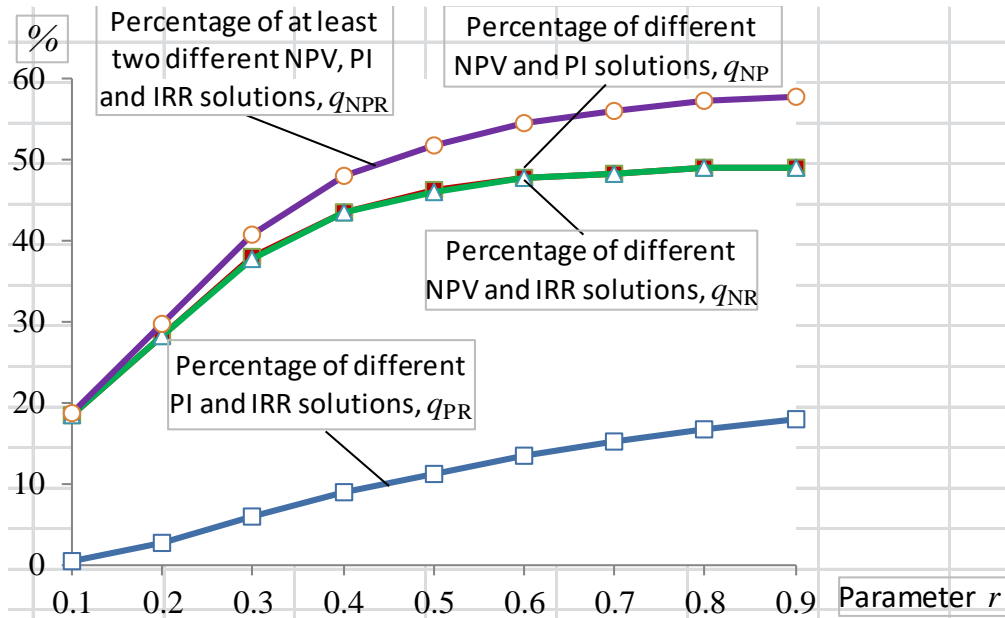
Figure 5. Percentages $q_{NP}(r)$, $q_{NR}(r)$, $q_{PR}(r)$ and $q_{NPR}(r)$.

Table 4

The ranges of values for the four dependences on parameter r at disc. rate $d \in [0.05; 0.14]$, %

Indicators	$q_{NP}(r)^1$	$q_{NR}(r)^2$	$q_{PR}(r)^3$	$q_{NPR}(r)^4$
Minimum of $q(r)$	13.4	13.4	0	13.4
Maximum of $q(r)$	49.2	49.4	19.1	58.7

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI, and IRR solutions.

The group of alternatives a5 - dependence on v. Initial data: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 = 5$; $I_1 = 1000$, $I_2 = 500$; $r = 0.2$; $v = \{0.1, 0.2, 0.3, ..., 0.9\}$. The obtained dependences $q_{NP}(v)$, $q_{NR}(v)$, $q_{PR}(v)$ and q_{NPR} at $d = 0.08$ are shown in Figure 6.
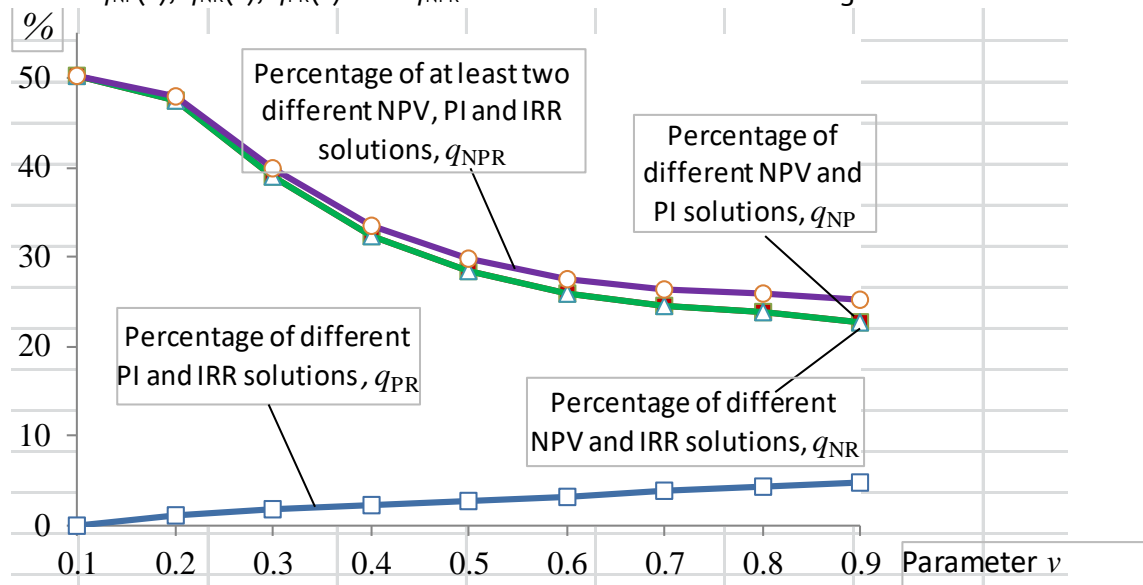

Figure 6. Percentages $q_{NP}(v)$, $q_{NR}(v)$, $q_{PR}(v)$ and $q_{NPR}(v)$.

According to Figure 6, three of the four dependences, namely $q_{NP}(v)$, $q_{NR}(v)$ and $q_{NPR}(v)$, are decreasing, and the $q_{PR}(v)$ one is slightly increasing on v. At the same time, at $v \in [0.1;$

0.2] take place $q_{NP}(v) \approx q_{NR}(v) \approx q_{NPR}(v)$, and at $v > 0.1$ the discrepancy between $q_{NP}(v) \approx q_{NR}(v)$ and $q_{NPR}(v)$ is slightly increasing, but is relatively small. The obtained ranges of values for the four dependences on v at $d \in [0.05; 0.14]$ are specified in Table 5.

The ranges of values for four dependences on parameter v at disc. rate $d \in [0.05; 0.14]$, %

Indicators	$q_{NP}(v)$[1]	$q_{NR}(v)$[2]	$q_{PR}(v)$[3]	$q_{NPR}(v)$[4]
Minimum of $q(v)$	20.6	20.2	0	21.7
Maximum of $q(v)$	50.01	50.03	5.6	50.4

[1] q_{NP} - percentage of different NPV and PI solutions; [2] q_{NR} - percentage of different NPV and IRR solutions; [3] q_{PR} - percentage of different PI and IRR solutions; [4] q_{NPR} - percentage of at least two different NPV, PI, and IRR solutions.

Based on data of Table 5, it can be concluded that, on average, there are a considerable number of cases when the use of at least two of the three examined indices leads to different solutions ($q_{NPR}(v) \in [21.7; 50.4]$%). The use of pairs of compared indices NP and NR also can lead to different solutions in a significant number of cases the respective range of values being approx. the same, but not exceeding 50 %. The narrowest range of values is that of $q_{PR}(v)$ equal to 5.56 %, and $q_{PR}(v) = 0$ at $\{v = 0.1, d \in [0.12, 0.14]\}$. Also, because of the smallest values of the percentage $q_{PR}(v) \in [0; 5.6]$%, from the three compared indices, the PI and IRR are the closest to each other.

The group of alternatives a6 - dependence on $d+$ (on d when $D_1 = D_2$, I_1 and I_2 are generated randomly). Initial data: $d = \{0.05, 0.06, 0.07, ..., 0.14\}$; $D_1 = D_2 \in [1; 10]$; $I_1 \in [100; 1000]$, $I_2 \in [100; 1000]$; $r = 0.2$; $v = 0.5$. The dependences $q_{NP}(d+)$, $q_{NR}(d+)$, $q_{PR}(d+)$ and $q_{NPR}(d+)$ are shown in Figure 7.

Similar to the group of alternatives a1 (dependence on d), for group a6 all four dependences are decreasing on d, and the ones for the pairs $q_{NP}(d+)$ and $q_{NR}(d+)$ practically coinciding. At the same time, the discrepancy between percentages $q_{NP}(d+) \approx q_{NR}(d+)$ and $q_{NPR}(d+)$ is slightly decreasing on d. Also, take place the relations $q_{PR}(d+) < q_{NP}(d+) \approx q_{NR}(d+) < q_{NPR}(d+)$. The obtained ranges of values for the four dependences at $d \in [0.05; 0.14]$ are specified in Table 6.
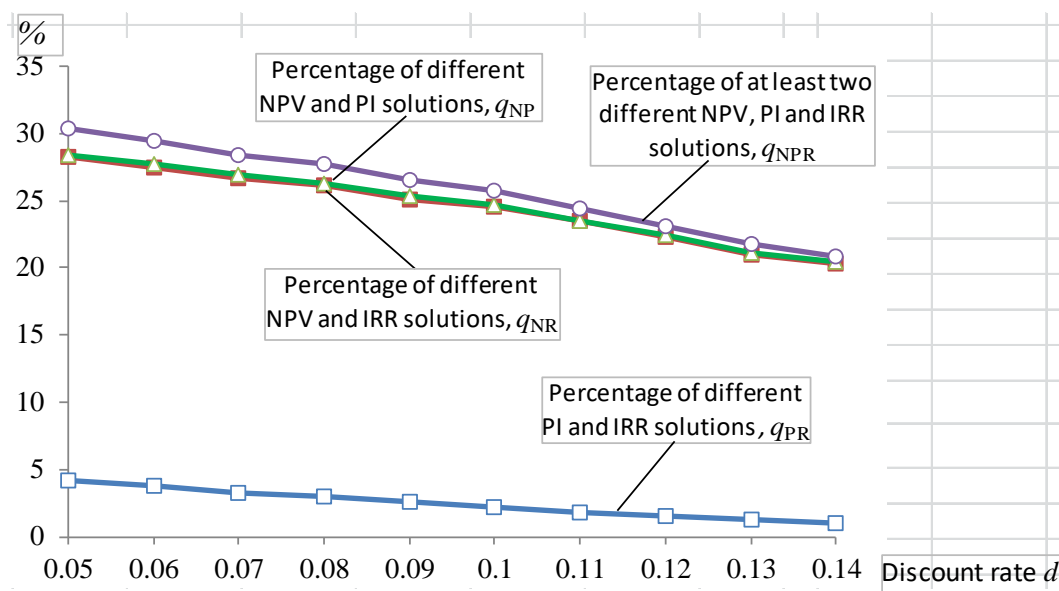

Figure 7. Percentages $q_{NP}(d+)$, $q_{NR}(d+)$, $q_{PR}(d+)$ and $q_{NPR}(d+)$.

The ranges of values for the four dependences of case $d+^5$ at disc. rate $d \in$ [0.05; 0.14], %

Indicators	$q_{NP}(d+)$	$q_{NR}(d+)$	$q_{PR}(d+)$	$q_{NPR}(d+)$
Minimum of $q(d+)$	20.3	20.4	1.1	20.9
Maximum of $q(d+)$	28.2	28.4	4.2	30.4

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI, and IRR solutions; $d+$ - case when $D_1 = D_2$, I_1 and I_2 are generated randomly.

On average, there are a significant number of cases when the use of investigated pairs of indices leads to different solutions; for example $q_{NPR}(d+) \in$ [20.9; 30.4] %. The largest range of values is that of $q_{NPR}(d+)$ equal to 30.4 – 20.9 = 9.5%, and the narrowest range is that of $q_{PR}(d+)$ equal to 4.2 – 1.1 = 3.1%. From the three compared indices, the PI and IRR are the closest to each other: $q_{PR}(d+) \in$ [1.1; 4.2] %,

The group of alternatives a7 – general group (on d when $D_1 = D_2$, I_1, I_2, r and v are generated randomly). Initial data: d = {0.05, 0.06, 0.07, ..., 0.14}; $D_1 = D_2 \in$ [1; 10]; $I_1 \in$ [100; 1000], $I_2 \in$ [100; 1000]; $r \in$ [0.1; 0.9]; $v \in$ [0.1; 0.9]. Some of the obtained results of calculations for the four dependences at $d \in$ [0.05; 0.14] are systemized in Table 7.

On average, for the group of alternatives of initial data a7 the number of cases when the use of indices of researched pairs leads to different solutions is less than 35.7 %, and overall, that is when at least two of the three examined indices leads to different solutions is less than 40.7 %. The largest range of values is that of $q_{NPR}(d\cdot)$ equal to 40.7 – 37.9 = 2.8 % ($q_{PR}(d\cdot) \in$ [37.9; 40.7]%), and the narrowest range is that of $q_{NP}(d\cdot)$ equal to 34.7 – 33.4 = 1.3 %. As in previous six groups of alternatives, because of the smallest values of percentage $q_{PR}(d) \in$ [8.3; 11.0]%, from the three compared indices, the PI and IRR are the closest to each other.

Percentages for the four dependences of case $d\cdot^5$ at discount rate $d \in$ [0.05; 0.14], %

d	$q_{NP}(d\cdot)$[1]	$q_{NR}(d\cdot)$[2]	$q_{PR}(d\cdot)$[3]	$q_{NPR}(d\cdot)$[4]
0.05	34.68	35.74	10.95	40.69
0.06	34.68	35.56	10.55	40.40
0.07	34.51	35.38	10.09	39.99
0.08	34.51	35.28	9.85	39.82
0.09	34.17	35.09	9.47	39.36
0.1	34.02	34.68	9.33	39.01
0.11	34.03	34.85	9.31	39.10
0.12	33.54	34.15	8.89	38.29
0.13	33.40	34.28	8.65	38.19
0.14	33.41	34.03	8.32	37.88
Minimum of $q(d\cdot)$	33.40	34.03	8.32	37.88
Maximum of $q(d\cdot)$	34.68	35.74	10.95	40.69
Average of $q(d\cdot)$	34.09	34.91	9.54	39.27

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI and IRR solutions; $d\cdot$ - case when $D_1 = D_2$, I_1, I_2, r and v are generated randomly.

The obtained dependences $q_{NP}(d\cdot)$, $q_{NR}(d\cdot)$, $q_{PR}(d\cdot)$ and $q_{NPE}(d\cdot)$) are shown in Figure 8.
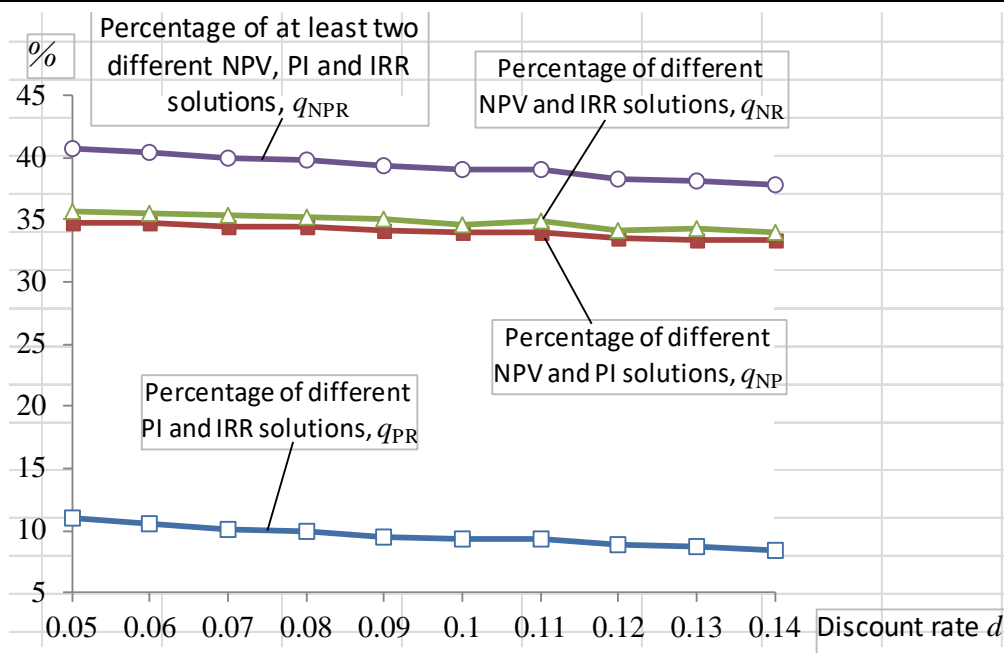
Figure 8. Percentages $q_{NP}(d\cdot)$, $q_{NR}(d\cdot)$, $q_{PR}(d\cdot)$ and $q_{NPRE}(d)$.

Like the groups of alternatives a1 (dependence on d) and a6 (dependence on $d+$), for the group a7 all four dependences are decreasing on d, but slightly than the $q_{PR}(v)$ for nominated two. At the same time, this is the only group of the seven examined for which clearly occurs $q_{NP}(d\cdot) < q_{NR}(d\cdot)$, and the discrepancy between $q_{NP}(d\cdot)$ and $q_{NPR}(d\cdot)$ as well as the one between $q_{NR}(d\cdot)$ and $q_{NPR}(d\cdot)$ are relatively large at $d \in [0.05; 0.14]$.

3.3. Generalization of the results of computer simulation

Figures 2-8 shows $4 \times 7 = 28$ dependences, of which 18 are decreasing, 6 are increasing, 3 initially are decreasing and after are increasing, and 1 is, practically, invariable. So, dependencies $q_{NP}(\cdot)$, $q_{NR}(\cdot)$, $q_{PR}(\cdot)$ and $q_{NPR}(\cdot)$ on d (Figure 2), on I_2 (Figure 4), on v (Figure 6), on $d+$ (Figure 7) and on $d\cdot$ (Figure 8) are decreasing or slightly decreasing, except that:

a) $q_{PR}(I_2)$ is, practically, invariable (Figure 4);

b) $q_{PR}(v)$ is slightly increasing (Figure 6).

Are increasing also dependences: $q_{PR}(D)$ (Figure 3); $q_{NP}(r)$, $q_{NR}(r)$, $q_{PR}(r)$ and $q_{PR}(r)$ (Figure 5). At the same time, dependences $q_{NP}(D)$, $q_{NR}(D)$ and $q_{NPR}(D)$ are initially decreasing and after increasing (Figure 3).

By pairs, in groups a1-a6 of alternatives of initial data, the dependences $q_{NP}(\cdot)$ and $q_{NR}(\cdot)$ practically coincide, and in group a7 they are very close to each other. Relatively close to them is also the dependence $q_{NPR}(\cdot)$. With refer to percentages $q_{PR}(\cdot)$, usually these are considerable smaller than the $q_{NP}(\cdot)$, $q_{NR}(\cdot)$ and $q_{NPR}(\cdot)$ ones. Thus, from the NPV, PI and IRR indices, the last two are the closest to each other regarding the solutions of comparing the efficiency of projects obtained. A comparative analysis of the range of values for the four percentages can be done based on data of Table 8.

So, at used seven groups of alternatives of initial data, the average percentage of cases with different solutions for all three pairs of indices usually is considerable, namely: $q_{NP}(\cdot) \in$ [0; 50.01] %, $q_{NR}(\cdot) \in$ [1.26; 50.03] % and $q_{PR}(\cdot) \in$ [0; 19.11] %. Also, the average percentage $q_{NPR}(\cdot)$ of cases with different solutions, when using of at least two of the three examined indices (NPV, PI and IRR), is in the range of values [1.26; 58.67] %. The overall size of the value range is approx.: 50 % for $q_{NP}(\cdot)$, 49 % for $q_{NR}(\cdot)$, 19 % for $q_{NR}(\cdot)$ and 57 % for $q_{NPR}(\cdot)$.

At the same time, there are categories of sets of initial data when examined indices in pairs always lead to the same solution, including the pairs:

Characteristics of the range of values for the four dependencies, %

Indicators		$q_{NP}(\cdot)$[1]	$q_{NR}(\cdot)$[2]	$q_{PR}(\cdot)$[3]	$q_{NPR}(\cdot)$[4]
Minimum of	$q(d)$[5]	21.60	21.60	1.30	22.20
	$q(D)$[6]	20.32	21.05	0	21.14
	$q(I_2)$[7]	0	1.26	1.24	1.26
	$q(r)$[8]	13.36	13.36	0	13.36
	$q(v)$[9]	20.58	20.20	0	21.71
	$q(d+)$[10]	20.31	20.43	1.07	20.90
	$q(d\cdot)$[11]	33.40	34.03	8.32	37.88
Overall minimum		0	1.26	0	1.26
Maximum of	$q(d)$	32.10	32.10	3.84	34.00
	$q(D)$	47.67	47.57	7.06	51.15
	$q(I_2)$	48.34	48.31	3.89	50.25
	$q(r)$	49.22	49.35	19.11	58.67
	$q(v)$	50.01	50.03	5.56	50.35
	$q(d+)$	28.22	28.40	4.16	30.39
	$q(d\cdot)$	34.68	35.74	10.95	40.69
Overall maximum		50.01	50.03	19.11	58.67
Overall range value		50.01	48.77	19.11	57.41

[1]q_{NP} - percentage of different NPV and PI solutions; [2]q_{NR} - percentage of different NPV and IRR solutions; [3]q_{PR} - percentage of different PI and IRR solutions; [4]q_{NPR} - percentage of at least two different NPV, PI and IRR solutions; [5]d – discount rate; [6]D – projects lifetime; [7]I_2 – project 2 investments; r – parameter for IRR value; v – parameter for the variation of CF_t values; $d+$ - case when $D_1 = D_2$, I_1 and I_2 are generated randomly; $d\cdot$ - case when $D_1 = D_2$, I_1, I_2, r and v are generated randomly.

- {NPV, PI} for group a3 (dependence on I_2) at $I_1 = I_2 = 1000$, that is obvious;
- {PI, IRR} for group a2 (dependence on D) at $D = 1$, for group a4 (dependence on r) at $\{r = 0.1, d = 0.14\}$ and for group a5 (dependence on v) at $\{v = 0.1, d \in [0.12, 0.14]\}$.

But there were not identified such categories of sets of initial data when using the NPV and IRR indices or, as a result, all three exam ined indices (NPV, PI and IRR) together.

It is useful also to mention that, based on group a7 of alternatives of initial data (general group – dependence on d when $D_1 = D_2$, I_1, I_2, r and v values are generated randomly), the average percentage of cases with different solutions is approx. (in the increasing order): 9.1 % for $q_{PR}(\cdot)$, 34.1% for $q_{NP}(\cdot)$, 34.9 % for $q_{NR}(\cdot)$ and 39.3 % for $q_{NPR}(\cdot)$ (see Table 7). Thus, on average, the solutions of comparing the efficiency of projects obtained, when using the NPV, PI and IRR indices, does not coincide in more than 1/3 of cases.

4. Conclusions

To research comparatively by computer simulation the NPV, PI and IRR indices, used when selecting investment i-projects with equal lives, a model of comparative analysis of projects is defined and the SIMINV application is made up.

Each of the two compared projects is characterized by: discount rate d, duration D, volume of investment I and cash flows CF_t, t = 1, 2, ..., D. From these characteristics, only the

values of *d* and *D* are common for both projects. The other characteristics in some cases have fixed value and in other cases are generated randomly, in such a way forming seven groups of alternatives of initial data.

By computer simulation, the percentages of cases when the solutions, obtained using indices of each of the pairs {NPV, PI} – q_{NP}, {NPV, IRR} – q_{NR}, {PI, IRR} – q_{PR} or at least two of the NPV, PI and IRR indices – q_{NPR}, does not coincide is determined. These results complement, to some extent, the known theoretical ones in the domain.

So, for all seven groups of alternatives of initial data are determined the quantitative values and the character of dependencies $q_{NP}(\cdot)$, $q_{NR}(\cdot)$, $q_{PR}(\cdot)$ and $q_{NPR}(\cdot)$. There are categories of sets of initial data when examined indices in pairs always lead to the same solution. But there were not identified such categories of sets of initial data when using the NPV and IRR indices or, as a result, all three examined indices (NPV, PI and IRR) together.

The average percentage of cases, for which the obtained solutions does not coincide, is of approx. (in the increasing order): 9.1 % for $q_{PR}(\cdot)$, 34.1% for $q_{NP}(\cdot)$, 34.9 % for $q_{NR}(\cdot)$ and 39.3 % for $q_{NPR}(\cdot)$, being considerable. Thus, from the NPV, PI and IRR indices, the last two are the closest to each other regarding the solutions of comparing the efficiency of projects obtained. Also, on average, the solutions of comparing the efficiency of projects, obtained when using the NPV, PI and IRR indices, does not coincide in more than 1/3 of cases.

Conflicts of Interest. The authors declare no conflict of interest.

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