Tuning the PID Controller to the Model of Object with Inertia Second Order According to the Maximum Stability Degree Method with Iteration

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***Abstract* – In this paper there was elaborate the procedure for tuning the PID controller to the object with inertia of the second order according to the maximum degree of stability method with iterations. The characteristic equation of the closed system is determined. By derivative operations on the unknown variable of the maximum degree, analytical expressions of the parameters of the PID controller are obtained as functions of the parameters of the object and the degree of stability. By graph-analytical procedures these functions are constructed by varying the argument. By procedures with iterations on these curves for the same value of the argument, sets of values of the parameters controller are chosen. The synthesized system is simulated and the system performances are evaluated. Two examples of objects with inertia less and greater than one according to the proposed method and the pole-zero allocation method by simulating on the computer system were examined. The parameters of the objects were varied and the robustness of the system at the unit step reference action was verified. The advantages of the proposed method are highlighted by reduced calculations and minimal time, which lead to the simplification of the procedure for tuning the controller for these objects.**

Cuvinte cheie: *modelul obiectului cu inerţie; regulator PID; acordarea regulatorului; performanţele sistemului.*

Keywords: *model of object with inertia; PID controller; tuning algorithms; performances of the system.*

I. INTRODUCTION

In the automation of various industrial processes, it is necessary to determine the mathematical models with the constant parameters attached to these processes. By the applying analytical or experimental identification methods, based on the operational requirements of these processes, there are obtained the mathematical models, that can have high order inertia. The procedures for synthesis the control algorithms to these types of models of objects could become difficult, being accompanied by a larger volume of calculations.

Based on these considerations, it is rational to obtain a mathematical model with a low order inertia, with constant parameters and then the control algorithm with reduced order can be synthesized.

In this paper the mathematical model is used (for example, DC motor, thermal process etc.) with inertia second order with known parameters with two poles allocated in the left half of the complex plane, which has a broad application in practice [1-4], and it is described by the following transfer function:

$$H(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)} = \frac{k}{a_0 s^2 + a_1 s + a_2}, \qquad (1)$$

where k is the transfer coefficient of the model, T_1, T_2 – time constants, and $a_0 = T_1 T_2$, $a_1 = T_1 + T_2$, $a_2 = 1$.

Actually, there are developed many synthesis methods for tuning the typical controllers, which can be applied to the model of objects with high inertia order. The practical aspects of realization these algorithms lead to the sophisticated structures, which reduce the reliability of the automatic control system [4].

In the practice of automation, the various industrial processes, there has a wide application the controllers with fixed structure as PID type [5-10].

For the model of the control object (1) for the synthesis of the PID type control algorithms, the frequency method and the poly-zero allocation method can be applied. The application of the frequency methods [11, 12] are accompanied by calculations in the frequency domain and graphical constructions that lead to difficulties in the synthesis of the adjustment algorithms.

There are a lot of tuning methods the PID controller to the model of object with inertia second order [1-4]. From the existing methods of tuning the PID controller some of them present the difficult procedures of tuning, and other methods guarantee stability of the automatic control system, but do not guarantee the quality of the operating regime.

The pole-zero allocation method is an analytical method, which based on the model of control object (1), the imposed performance to the automatic control system the settling time t_r and overshoot d, and the model of the control algorithm PI and PID with unknown tuning parameters. It is built the polynomial equation of the closed loop automatic system, from which it is obtained the system of diofantic equations. Solving this system of equations are determined the tuning parameters which satisfy the stability requirements and the imposed performance to the system. If the imposed performance are not satisfied, in the polynomial equation are introduced additional poles-zeros, which are allocated in the left half of the complex plane, placed as far as possible from the main poles until all requirement performance are achieved [3].

The method of pole-zero allocation in the complex plan for tuning the control algorithms is a well known method, which is described in [3] and will be applied for tuning the PID control algorithm.

From these considerations, there was developed the procedure for tuning the control algorithms to the model of object (1) by the maximum stability degree (MSD) method with iterations.

Thus, the procedure for tuning the control algorithm is the procedure with iterations.

The maximum stability degree (MSD) method in the classical version it is not applied in case of tuning the PID controller to the model of object (1) [13]. The procedure of tuning the PI and PID control algorithms is reduced to the formation of the characteristic equation of the closed loop system, it is introduced the notion of stability degree in the characteristic equation, as a new unknown variable, and by derivation operations of its, it is obtained a system of algebraic equations, from which the maximum stability degree and tuning parameters can be calculated.

In this paper it was proposed to use the maximum stability degree method with iterations [14, 15].

For two examples of the control model of object (1), there is analyzed the procedures of tuning the PI and PID algorithms according to the applied methods, the system is simulated and the performance of the automatic control system are analyzed.

There was analyzed the efficiency of tuning the typical PID algorithm based on the performance and robustness of the automatic control system, when on the system acts the disturbance signal as step signal and when it is the variation of the model object parameters k, T_1, T_2 with ± 50% from the nominal values of the object model.

II. ALGORITHM OF TUNING THE PI AND PID CONTROLLERS

In the study it is used the block scheme of the automatic control system that consists from the controller with transfer function $H_R(s)$ and model of object with transfer function $H(s)$, presented in the Fig. 1.

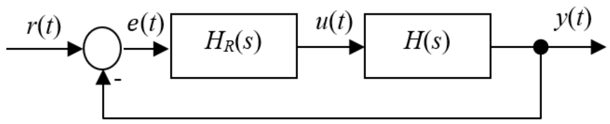

Fig. 1. The structural scheme of the automatic control system.

The PI and PID control algorithms in the standard form are described by the transfer functions in the following form:

$$H_{PI}(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s}, \qquad (2)$$

$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \qquad (3)$$

where k_p, k_i are the tuning parameters of the PI control algorithm and k_p, k_i, k_d – are the tuning parameters of the PID control algorithm.

It will be performed the tuning of the control algorithms (2) and (3) to the model of object (1) by the maximum stability degree method with iterations [14, 15].

Bellow it is presented the maximum stability degree method with iterations for determination the tuning parameters k_p, k_i, k_d of the PI and PID controllers to the model of object (1).

For the automatic control system with PI algorithm are obtained three expressions for calculation the tuning parameters [7, 8]:

$$k_p = \frac{1}{k}(-3a_0J^2 + 2a_1J - a_2) =$$
$$= \frac{1}{k}\left(\frac{a_1^2}{3a_0} - a_2\right) = f_p(J, k, a_0, a_1, a_2) = f_p(J), \qquad (4)$$

$$k_i = \frac{1}{k}(-2a_0J^3 + a_1J^2) = \frac{1}{k}(a_0J^3 - a_1J^2 + a_2J) +$$
$$+ k_pJ = f_i(J, k, a_0, a_1, a_2) = f(J), \qquad (5)$$

$$J = \frac{a_1}{3a_0}. \qquad (6)$$

Further, using the expression (6), based on the relations (4)-(5) can be calculated the tuning parameters of the PI algorithm, and thus the synthesis problem is solved.

For the automatic control system with the PID algorithm, are obtained three expressions for calculation the tuning parameters:

$$k_p = \frac{1}{k}(-3a_0J^2 + 2a_1J - a_2) + 2k_dJ =$$
$$= \frac{1}{k}\left(3a_0J^2 - a_2\right) = f_p(J, k, a_0, a_1, a_2) = f_p(J), \qquad (7)$$

$$k_i = \frac{1}{k}(a_0J^3 - a_1J^2 + a_2J) - k_dJ^2 + k_pJ =$$
$$= \frac{a_0}{k}J^3 = f_p(J, k, a_0, a_1, a_2) = f_i(J), \qquad (8)$$

$$k_d = \frac{1}{k}\left(3a_0J - a_1\right) = f_p(J, k, a_0, a_1, a_2) = f_d(J). \qquad (9)$$

A possible way for determination the tuning parameters of the PID algorithm is reduced to the condition that the coefficient $k_d > 0$ is necessary that for the maximum stability degree to achieve inequality:

$$J > \frac{a_1}{3a_0}. \qquad (10)$$

Further, using the expression (10), based on the relations (7)-(9) can be calculated the tuning parameters of the PID algorithm, and thus the synthesis problem is solved.

To optimize the performance of the automatic control system, it is proposed the following procedure.

Expressions (4)-(5) and (7)-(9) for calculation the tuning parameters of the PI and PID control algorithms are presented as functions $k_p = f_p(J)$, $k_i = f(J)$, $k_d = f(J)$ on variable J, which presents the maximum stability degree of the designed system.

At the variation of the J as independent variable it is calculated and constructed the curves (4)-(5) and (7)-(9)

$k_p = f_p(J)$, $k_i = f(J)$, $k_d = f(J)$ for the PI and PID algorithms.

For these curves $k_p = f_p(J)$, $k_i = f(J)$, $k_d = f(J)$ are chosen the values of the tuning parameters of the PI and PID controller J_i – k_{pi}, k_{ii}, k_{di} and computer simulation is done, are obtained the transient processes based on these are determinate the performance of the automatic control system, which would satisfy the imposed requirements of the system.

For two examples of models of control objects (1), the procedures for tuning the PI and PID algorithms are analyzed according to the maximum degree of stability method with iterations and the poly-zero method, the system is simulated and the performances of the designed system are analyzed.

In order to verify the obtained results in case of tuning the PI and PID algorithms to the model of object (1) by the pole-zero allocation method and maximum stability degree method with iterations it is presented a calculation two examples.

Example 1. It is given the model of object with inertia second order with known parameters: transfer coefficient $k = 0.2$ and time constants $T_1 = 0.5$ s, $T_2 = 0.2$ s, $a_0 = T_1T_2 = 0.5 * 0.2 = 0.1$ s^2, $a_1 = T_1 + T_2 = 0.5 + 0.2 = 0.7$ s, $a_2 = 1$, which are described by the transfer function model [1, 3, 4]:

$$H(s) = \frac{k}{(T_1s+1)(T_2s+1)} = \frac{k}{a_0s^2 + a_1s + a_2} = \frac{2}{(s+2)(s+5)} = \\ = \frac{0.2}{0.1s^2 + 0.7s + 1} = \frac{2}{s^2 + 7s + 10} = \frac{B(s)}{A(s)}. \qquad (11)$$

For the imposed performance of the automatic control system with settling time $t_r \le 2$ s and overshoot $d \le 10$%, it is synthesized the control algorithm PI and PID respectively to the model of object (1) by the maximum stability degree method and pole-zero allocation method.

There were done the calculation for tuning the PI and PID controller by the maximum stability degree method using relations (4)-(5) and (7)-(9) to the model of object (1) with known parameters and the obtained curves are presented in the Figure 2, *a*, *b*.

The automatic control system with the PI and PID controller was simulated (Figure 3) and the obtained transient processes are presented in the Figure 4, and the performance settling error $e = 5$ %, settling the time t_r and overshoot d of the system are given in the Table I (curve numbering corresponds to row number 5 - PID controller (curve 1) and 12 – PI controller (curve 2) from Table I) and the performance of the automatic control system are given in the rows 5 and 12 from Table I.

Using the procedure for calculation the tuning parameters by the pole-zero allocation method, according to the imposed performance to the system settling the time $t_r = 2$ s and overshoot $d = 10$ %, there are determinate the dominant poles of the characteristic polynomial which have values $s_{1,2} = -2 \pm j2.66$, and it is designed the controller PID which provide the settling error equals to zero to the references and perturbation signal as unit step signal.

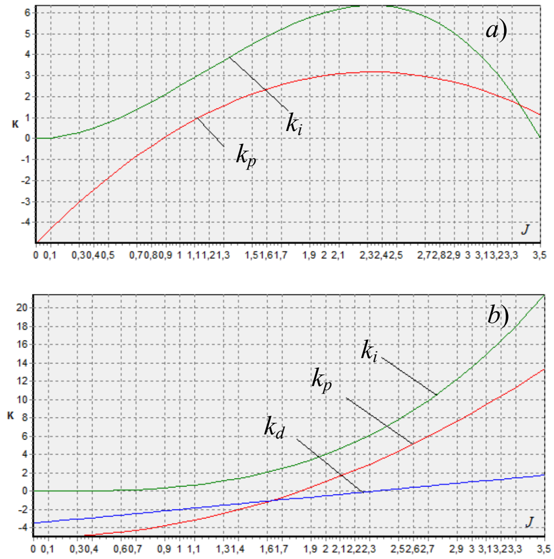

Fig. 2. Dependencies of the parameters $k_p = f_p(J)$, $k_i = f(J)$, $k_d = f(J)$ by degree of stability: *a*) curves of PI controller; *b*) curves of PID controller.

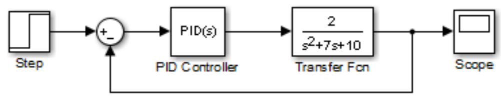

Fig. 3. Simulation diagram of the control system.

According to the pole-zero allocation method it is constructed the desired characteristic polynomial of the designed system which has the order $r = 2n = 4$, composed from dominant poles $s_{1,2} = -2 \pm j2.66$ and two additional poles $s_{3,4} = -10$, allocated at the real negative semiaxis as far as possible from the dominant poles, so that the controller to be physically realizable and to assure the imposed performance of the system and that has the form:

$$P_c(s) = (s + s_1)(s + s_2)(s + s_3)(s + s_4) = \\ = (s + 2 - j2.66)(s + 2 + j2.66)(s + 10)^2 = \\ = (s^2 + 4s + 11)(s + 10)^2 = \\ = s^4 + 24s^3 + 191s^2 + 620s + 1100 . \qquad (12)$$

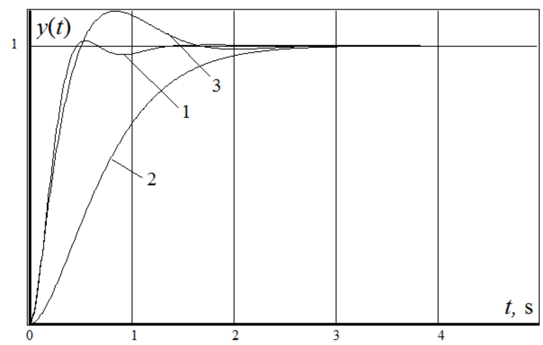

Fig. 4. Transient processes of the automatic control system (ex.1).

It is chosen the PID control algorithm with the real derivative component in the following form:

$$H_R(s)=\frac{q_2s^2+q_1s+q_0}{s(p_1s+p_0)}=\frac{q_2s^2+q_1s+q_0}{p_1s^2+p_0s}=\frac{Q(s)}{P(s)}, \quad (13)$$

where p_1, p_0, q_2, q_1, q_0 are unknown tuning parameters, which will be calculated.

TABLE I.
PERFORMANCE OF THE SYSTEM FOR EXAMPLE 1

No. iter.	Con. type	Tuning parameters					Performance of the control system			
		J	k_p	k_i	T_i, s	k_d, s	t_c, s	d, %	t_r, s	l
1	PID	2.4	3.64	6.91	0.14	0.1	1.6	-	1.6	-
2	PID	2.5	4.38	7.81	0.12	0.2	1.5	-	1.5	-
3	PID	3.0	8.50	13.5	0.11	1.0	1.0	-	1.0	-
4	PID	3.5	13.3	21,4	0.04	1.7	0.4	-	0.4	-
5	PID	3.6	14.4	23.3	0.04	1.9	0.3	-	0.3	-
6	PID	3.7	15.5	25.3	0.03	2.0	0.3	6.50	0.5	1
7	PID	3.8	16.6	27.4	0.03	2.2	0.3	9.27	0.6	1
8	PID	4.0	19.0	32.0	0.03	2.5	0.2	14.6	0.9	1
9	PID	3.6	14.4	23.3	0.04	1.9	0.49/ 0.25	5.41/ 4.0	0.87/ 0.25	1/0
10	PID	3.6	14.4	23.3	0.04	1.9	0.44/ 0.85	9.3/ 0,0	0.87/ 0.85	1/0
11	PID	3.6	14.4	23.3	0.04	1.9	0.27/ 1.25	14.6/ 0.0	0.85/ 1.25	1/0
12	PI	2.3	3.17	6.35	0.15	-	1.8	-	1.8	-
13	PID						0.45	12.5	1.3	1
14	PID						0.54/ 0.37	20.9/ 0.0	1.6/ 0.37	1/0
15	PID						0.49/ 0.48	20.9/ 5.6	1.44/ 1.13	1/1
16	PID						0,31/ 0.8	12.7/ 10.3	1.01/ 1.94	1/1

It is constructed the polynomial equation of the control system in the form:

$$1+H(s)H_R(s)=1+\frac{B(s)}{A(s)}\frac{Q(s)}{P(s)}=$$
$$=A(s)P(s)+B(s)Q(s)=P_c(s)\,, \quad (14)$$

or in the form:

$$(s^2+7s+10)(p_1s^2+p_0s)+2(q_0+q_1s+q_2s^2)=$$
$$=s^4+24s^3+191s^2+620s+1100\,,$$
$$p_1s^4+s^3(7p_1+p_0)+s^2(10p_1+7p_0+2q_2)+$$
$$+s(10p_0+2q_1)+2q_0=$$
$$=s^4+24s^3+191s^2+620s+1100\,.$$

After some transformations of polynomial expression and equalization of the coefficients with same powers from the left and right side it is obtained the matrix relationship:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 & 0 \\ 10 & 7 & 2 & 0 & 0 \\ 0 & 10 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} p_1 \\ p_0 \\ q_2 \\ q_1 \\ q_0 \end{bmatrix}=\begin{bmatrix} 1 \\ 24 \\ 191 \\ 620 \\ 1100 \end{bmatrix},$$

from which are obtained the tuning parameters $p_1=1$, $p_0=17$, $q_2=31$, $q_1=225$, $q_0=550$ of the PID algorithm with transfer function:

$$H_R(s)=\frac{Q(s)}{P(s)}=\frac{q_2s^2+q_1s+q_0}{s(p_1s+p_0)}=\frac{31s^2+225s+550}{s(s+17)}\,.$$

It was done the computer simulation (Figure 3) with the model of object (1) and the PID control algorithm (13), and the transient process is presented in the figure 4 (curve 3, which corresponds to row 13 from Table I, and the performances of the automatic control system are given in Table I (row 13).

Example 2. It is given the model of object with inertia second order with known parameters: transfer coefficient $k=0.2$, time constants $T_1=10$ s, $T_2=4$ s, $a_0=T_1T_2=$ $=10*4=40$ s^2, $a_1=T_1+T_2=10+4=14$ s, $a_2=1$, which are described by the transfer function [5-7]:

$$H(s)=\frac{k}{(T_1s+1)(T_2s+1)}=\frac{k}{a_0s^2+a_1s+a_2}=$$
$$=\frac{0.5}{40s^2+14s+1}=\frac{0.0125}{(s+0.1)(s+0.25)}=$$
$$=\frac{0.0125}{s^2+0.35s+0.025}=\frac{B(s)}{A(s)}, \quad (15)$$

For the imposed performance of the automatic system with settling time $t_r\le 2\,s$ and overshoot $d\le 10$ %, it is synthesized the control algorithm PI and PID to the model of object (1) by the maximum stability degree method and pole-zero allocation method.

There were done the calculation for tuning the PI and PID controller by the maximum stability degree method using relations (4)-(5) and (7)-(9) to the model of object (1) with known parameters and the obtained curves are presented in the Figure 5, *a*, *b*.

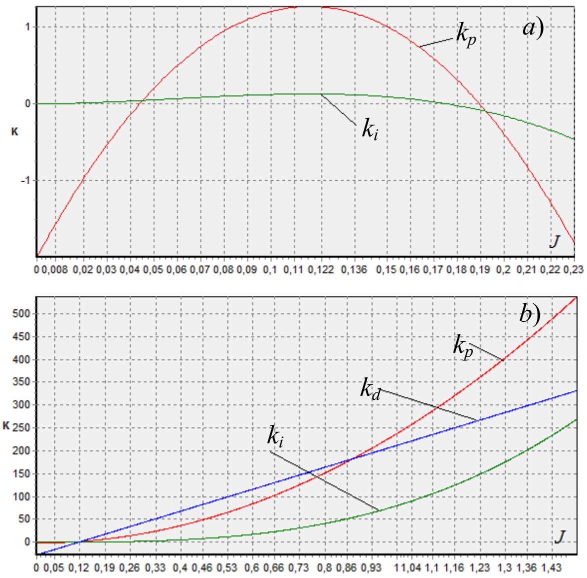

Fig. 5. Dependencies of the parameters $k_p=f_p(J)$, $k_i=f(J)$, $k_d=f(J)$ by degree of stability: *a*) curves of PI controller; *b*) curves of PID controller.

The automatic control system with the PI and PID controller was simulated (Figure 6) and the obtained transient processes are presented in the Figure 7, and the performance of the system are given in the Table II (curve numbering corresponds to row number 5 - PID controller (curve 1)) and the performance of the automatic control system are given in the row 5 from Table II.

Using the procedure for calculation the tuning parameters by the pole-zero allocation method, according to the imposed performance to the system settling the time $t_r = 2$ s and overshoot $d = 10\,\%$, there are determinate the dominant poles of the characteristic polynomial which have values $s_{1,2} = -2 \pm j2.66$, and it is tuned the controller PID which provides the settling error equals to zero to the references and perturbation signal as unit step signal.

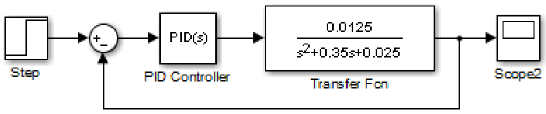

Fig. 6. Simulation diagram of the control system.

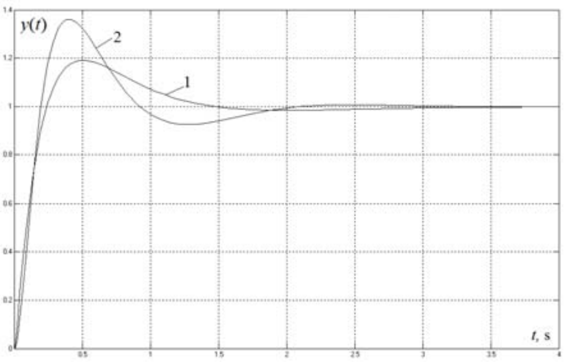

Fig. 7. Transient processes of the automatic control system.

According to the pole-zero allocation method it is constructed the desired characteristic polynomial of the designed system which has the order $r = 2n = 4$, composed from dominant poles $s_{1,2} = -2 \pm j2.66$ and two additional poles $s_{3,4} = -10$, allocated at the real negative semiaxis as far as possible from the dominant poles, so that the controller to be physically realizable and to assure the imposed performance of the system and that has the form:

$$P_c(s) = (s+s_1)(s+s_2)(s+s_3)(s+s_4) =$$
$$= (s+2-j2.66)(s+2+j2.66)(s+10)^2 =$$
$$= (s^2+4s+11)(s+10)^2 =$$
$$= s^4+24s^3+191s^2+620s+1100 .$$

It is chosen the PID control algorithm with the real derivative component in the following form:

$$H_R(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(p_1 s + p_0)} = \frac{Q(s)}{P(s)} . \qquad (16)$$

where p_1, p_0, q_2, q_1, q_0 are unknown tuning parameters, which will be calculated.

It is constructed the polynomial equation of the control system in the form:

$$1 + H(s)H_R(s) = 1 + \frac{B(s)}{A(s)}\frac{Q(s)}{P(s)} =$$

$$= A(s)P(s) + B(s)Q(s) = P_c(s) , \qquad (17)$$

or in the form:

$$(s^2+0.35s+0.025)(p_1 s^2 + p_0 s) =$$
$$= 0.0125(q_2 s^2 + q_1 s + q_0) =$$
$$= s^4+24s^3+191s^2+620s+1100 ,$$
$$p_1 s^4 + s^3(0.35p_1 + p_0) + s^2(0.025p_1 + 0.35p_0 +$$
$$+0.0125q_2) + s(0.0125p_0 + 0.0125q_1) + 0.0125q_0 =$$
$$= s^4+24s^3+191s^2+620s+1100 .$$

After some transformations of polynomial expression and equalization of the coefficients with same powers from the left and right side it is obtained the matrix relationship.

The relationship matrix it is presented below.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.35 & 1 & 0 & 0 & 0 \\ 0.025 & 0.35 & 0.0125 & 0 & 0 \\ 0 & 0.025 & 0 & 0.0125 & 0 \\ 0 & 0 & 0 & 0 & 0.0125 \end{bmatrix} \begin{bmatrix} p_1 \\ p_0 \\ q_2 \\ q_1 \\ q_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 24 \\ 191 \\ 620 \\ 1100 \end{bmatrix}$$

from which are obtained the tuning parameters $p_1 = 1$, $p_0 = 23.65$, $q_2 = 14616$, $q_1 = 49531$, $q_0 = 88000$ of the PID algorithm with transfer function:

$$H_R(s) = \frac{Q(s)}{P(s)} = \frac{q_2 s^2 + q_1 s + q_0}{s(p_1 s + p_0)} =$$
$$= \frac{14616 s^2 + 49531 s + 88000}{s(s+23.65)}$$

It was done the computer simulation (Figure 6) with the model of object (1) and the PID control algorithm (17), and the transient process is presented in the figure 6 (curve 2, which corresponds to row 10 from Table II, and the performances of the automatic control system are given in Table II (row 10).

TABLE II.
PERFORMANCE OF THE SYSTEM FOR EXAMPLE 2

No. iter.	Con. type	Tuning parameters					Performance of the control system			
		J	k_p	k_i	T_i, s	k_d, s	t_c, s	d, %	t_r, s	l
1	PID	1.0	238	80	0.01	212	0.62	16.09	2.92	1
2	PID	1.5	538	270	0.003	332	0.37	18.02	1.88	1
3	PID	1.9	864	549	0.002	428	0.29	18.34	1.48	1
4	PID	2.2	1160	682	0.001	500	0.24	18.7	1.27	1
5	PID	2.5	1498	1250	0.0008	572	0.2	20.0	1.09	1
6	PID	2.2	1498	1250	0.0008	572	0.33/ 0.15	23.64/ 10.2	1.51/ 0.8	1/1
7	PID	2.2	1498	1250	0.0008	572	0,8/ 0.15	13.3/ 9.36	0.99/ 0.79	1/1
8	PID	2.2	1498	1250	0.0008	572	0.8/ 0.4	13.33/ 27.6	0.99/ 1.75	1/1
9	PI	).15	1.0	0.09	11.1	-	60	-	60	-
10	PID						0.18	37.8	1.53	2
11	PID						0.23/ 0.11	46.0/ 33.8	1.89/ 0.59	2/1
12	PID						0.23/ 0.11	46.7/ 32.7	1.88/ 0.61	21
13	PID						0.13/ 0.28	34.5/ 53.0	0.66/ 3.78	1/4

There was varied the parameters T_1, T_2 and k of the model object in the system with PID controller tuned by the proposed method and pole-zero allocation method from examples 1 and 2 from the nominal values with ±50% and system performance are given by the bar in the Table I and II.

The proposed method:

Table I, row 9 – T_1, row 10 – T_2, row 11 – k.

Table II, row 6 – T_1, row 7 – T_2, row 8 – k.

The pole-zero method:

Table I, row 14 – T_1, row 15 – T_2, row 16 – k.

Table II, row 11 – T_1, row 12 – T_2, row 13 – k.

Analyzing the performance t_c rise time, over-shoot d and the settling the time t_r of the automatic system with the PID controller is tuned according to the MSD method (PID-MSD) (row 5 of Table I) and the system with the PID regulator is tuned according to the poly-zero method (PID- PZ)) for example 1 (row 5, 13, Table I) and example 2 (row 5, 10, Table II) of the models of the adjusting object the following conclusions are stated.

The performances of the system with the PID-MSD controller from example 1 falls to those imposed, and the performances of the system with the PID-PZ controller the settling time fall, and the overshoot is above the limit imposed on the design. In the case of example 2 for both methods the settling time of both systems falls within the imposed limit, and the overshoot exceeds the limit: for the PID-MSD system the overshoot is 2 times higher, and for the PID-PZ system - it is 3.7 times higher than the limit imposed on the system.

The automatic system with the PID-MSD controller with the object model of example 1 has higher performances compared to the system performance with the PID-PZ controller, the rise time t_c is reduced by 1.5 times, the overshoot by 1.25 times and the settling time t_r by 4.33 times.

The automatic system with the PID-MSD controller with the object model of example 2 has higher performances compared to the system performance with the PID-PZ controller, the overshoot is reduced by 1.89 times and the settling time t_r by 1.4 times.

The analysis of tuning the PI algorithm according to the MSD method to the object model of example 1 (time constants are subunits) shows that the parameters of the controller are optimal and the system performance satisfies the requirements imposed on the system. However, the system performance with the PI algorithm is much lower - the settling time is 4.76 times higher than the system settling time with the PID-MSD algorithm and 1.38 higher than the system with the PID-PZ controller.

The analysis of tuning the PI algorithm according to the MSD method to the object model of example 2 (time constants are superunit) shows that at the optimal parameters of the controller the system performances do not satisfy the requirements imposed on the system and, as a result, that the MSD method is not recommended for tuning the PI algorithm to the object (1) with superunit time constants.

III. CONCLUSIONS

Analyzing the results obtained by tuning the PI and PID algorithms to the model (1) according to the MSD and PZ methods of examples 1 and 2, we find:

- The procedure for tuning the PID algorithm by the MSD method is simpler and with a smaller volume of calculations compared to the PZ method.

- The performance of the automatic system with the PID-MSD controller is higher (the rise time t_c is lower by 1.1-1.5 times, the overshoot time from 1.25-1.89 and the settling time t_r is lower by 4.33 -1.4 times) compared to the performances system with the PID-PZ controller. The transient process of the system with the PID-MSD controller has lower overshoot compared to the transient process of the system with the PID-PZ controller. With the MSD method it is possible to obtain the damped transient oscillatory process of the system with higher performances in comparison with the performances of the system with the controller tuned after the PZ method.

- The system performance with the PI controller tuned according to the MSD method is determined by an aperiodic process that satisfies the required performances for the time constants are subunit, but they are reduced 4.76 times compared to the performances of the system with PID-MSD and PID-PZ system. For the case when the time constants of the model (1) are overunit, the MSD method is not recommended for the PI algorithm.

- The analysis of the evolution of the automatic system when changing the values of the parameters of the object k, T_1, T_2 by ± 50% from the nominal values it is found that the system performances with the PID-MSD controller are higher in comparison with the performances of the system with the PID-PZ controller: in example 1 in the MSD method the overshoot varies from 0-15% and the settling time varies 0.25-1.25 s, and at the PZ method the overshoot varies 0-21% and the settling time 0.37-1.94; in example 2, in the MSD method, the overshoot varies from 0-21% and the settling time varies from 0.8-1.75 s, and at the PZ method the overshoot varies from 33-53% and the settling time from 0.59 to 3.78 s.

- From the analysis of the degrees of stability of the PID-MSD and PID-PZ systems from both examples, it follows that the systems with the PID-MSD controller have a higher robustness of 1.64 times and 1.25 times respectively compared to the systems with the PID-PZ controller.

- The performance of the automatic control system with the PID controller tuned by the maximum stability degree method is higher (the rise time is with 1.21 times lower, overshoot equals to zero, the settling time is with 3.47 times lower, the transient process is aperiodic) in comparison with performance of the automatic control system with PID controller tuned by the pole-zero allocation method. Using the maximum stability degree method it is possible to obtain the oscillating transient process with higher performances in comparison with performance obtained in case of tuning controller by the pole-zero method (see Table I, row 13).

- The performance of the system with PI controller tuned by the maximum stability degree method are higher (the obtained performance of the automatic control system satisfy the imposed performance of the system) than the performance of the automatic control system with controller tuned by the pole-zero method (see Table I, rows 12 and 13).

- It was analyzed the evolution of the automatic control system for the case of variation the values of the object parameters k, T_1, T_2 with 50% from the nominal values (the rise time t_c varies from 2.95 to 3.3 times, in case of variation the k with 50%, and the settling time increases up to 2.2 times in case when the time constants are varying T_1, T_2 with 50% from the nominal values) and the action of the perturbation signal is as unit step signal.

- The system with the PID controller tuned by the maxim stability degree method is more robust in comparison to the system with controller tuned by the pole-zero method (the stability degree of the system is J=2.26, and the system with PID controller tuned by the pole-zero method has J=3.94 – 1.74 times farther from the imaginary axis).

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