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Linear stability interval for geometrical parameter of the Newtonian eight bodies problem

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Abstract

We consider the Newtonian restricted eight bodies problem with incomplete symmetry. The linear stability of stationary points of this problem are investigated by some numerical methods. For geometric parameter the intervals of stability and instability are found. All relevant and numerical calculations are done with the computer algebra system Mathematica.

Keywords: Newtonian problem; differential equation of motion; configuration; stationary points; linear stability.

1 Introduction

At present, qualitative studies of dynamical models of space are based on the search for exact particular solutions of differential equations of motion and subsequent analysis of their stability using the latest advances in computer mathematics. For this, it is required, first of all, to develop mathematical methods and algorithms for constructing exact partial solutions, since in the case of the Newtonian many bodies problem, for example, the number of solutions found is very limited.

Most of the exact solutions found for the Newtonian n-bodies problem belong to the class of so-called homographic solutions, the sufficient conditions for their existence were obtained by A. Wintner in the first half of the twentieth century, and the necessary conditions were formulated later by E. A. Grebenikov (see [3]).

The research method is based on the application of the analytic and qualitative theory of differential equations, the stability theory of Lyapunov-Poincaré, and also on the use of the capabilities of modern computer algebra systems for performing numerical calculations, processing symbolic information, and visualizing the obtained results.

2 Description of the configuration

We will study a particular case of the n-bodies problem describing in a non-inertial space P_0xyz the motion of seven bodies P_0, P_1, P_2, P_3, P_4, P_5, P_6, with the masses m_0, m_1, m_2, m_3, m_4, m_5, m_6, which attract each other in accordance with the law of universal attraction. We will investigate the planar dynamic pattern formed by a square in the vertices of which the points P_1, P_2, P_3, P_4, are located, the other two points P_5, P_6, having the masses $m_5 = m_6$ are on the diagonal P_1P_3 of the square at equal distances from point P_0, in around which this configuration rotates with a constant angular velocity ω which is determined from the model parameters. We can assume that $P_1(1,1)$, $P_2(-1,1)$, $P_3(-1,-1)$, $P_4(1,-1)$, $P_5(\alpha,\alpha)$, $P_6(-\alpha,-\alpha)$, $f = 1, m_0 = 1$, $m_5 = m_6$. Then out of the differential equations of the motion we obtain the existence conditions of this configuration:

$$\begin{cases} m_1 = m_3,\, m_2 = m_4 = f_1\left(m_1, \alpha\right), \\ m_5 = m_6 = f_2\left(m_1, \alpha\right),\, \omega^2 = f_3\left(m_1, \alpha\right). \end{cases} \tag{1}$$

Intervals of admissible values for the parameter α are determined by the conditions

$$m_2 = m_4 > 0; m_5 = m_6 > 0; \omega^2 > 0. \tag{2}$$

It is known that this dynamic model generates a new problem – the restricted problem of eight bodies. It will be studied the motion of the body P with a infinitely small mass (the so-called passive gravitational body) in the gravitational field by the given seven bodies.

Differential equations that describe motion of the body $P(x;y;z)$ which gravitates passively in the field of the other seven bodies in the rotating space have the form (see [2]):

$$\begin{cases} \dfrac{d^2X}{dt^2} - 2\omega\dfrac{dY}{dt} = \omega^2 X - \dfrac{fm_0X}{r^3} + \dfrac{\partial R}{\partial X}, \\ \dfrac{d^2Y}{dt^2} + 2\omega\dfrac{dX}{dt} = \omega^2 Y - \dfrac{fm_0Y}{r^3} + \dfrac{\partial R}{\partial Y}, \\ \dfrac{d^2Z}{dt^2} = -\dfrac{fm_0Z}{r^3} + \dfrac{\partial R}{\partial Z}, \end{cases} \tag{3}$$

where

$$\begin{cases} R = f\sum\limits_{j=1}^{6} m_j \left(\dfrac{1}{\Delta_{kj}} - \dfrac{XX_j + YY_j + ZZ_j}{r_j^3}\right), \\ \Delta_j^2 = (X_j - X)^2 + (Y_j - Y)^2 + (Z_j - Z)^2, \\ r_j^2 = X_j^2 + Y_j^2 + Z_j^2,\ r^2 = X^2 + Y^2 + Z^2, \\ j = 1, 2, ..., 6, \end{cases} \tag{4}$$

$(X_j; Y_j; Z_j = 0)$ are the respective coordinates of the bodies P_1, P_2, P_3, P_4, P_5, P_6 and are determined by the conditions of existence of the studied configuration.

3 Determination of stationary points

Using the graphical possibilities of the system Mathematica, the stationary points of the system (3) were determined. For this we use an algorithm similar to the algorithms from [1] (see [3] for other configurations). For concrete values of m_1 and α we obtain concrete stationary points. Their linear stability is studied.

Theorem. *There are values of the parameters m_1 and α for which the bisectorial stationary points in the restricted eight bodies problem are stable in the first approximation.*

4 Determination of the admissible variations intervals for the parameters

Moreover, we obtain that only for $0.85812 < \alpha < 0.85854$ and $m_1 = 0.01$ there are stationary points in the research problem that are linearly stable.

5 Concluding remarks

We have determined sufficient existence conditions of configuration describing the restricted Newtonian eight bodies problem. We have used some built in functions of the Mathematica programming environment in order to determine the stationary points. Their linear stability has been studied. It has been demonstrated that there are values of the parameters m_1 and α for which the bisectorial stationary points are stable in the first approximation. Intervals of stability and instability for geometric parameter are found.

References

[1] E. Cebotaru. *The application of Mathematica to research the restricted eight bodies problem*. Computer Science Journal of Moldova, vol. 26, no. 2(77), 2018, pp. 182-189.

[2] E. Cebotaru. *On the restricted eight bodies problem*. Romai Journal, vol. 14, no. 1, 2018, pp. 43–62.

[3] E. A. Grebenikov. *On a mathematical problems of homographic dynamics*. Moscow: MAKS Press, 2010, 253 p. (in Russian)

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